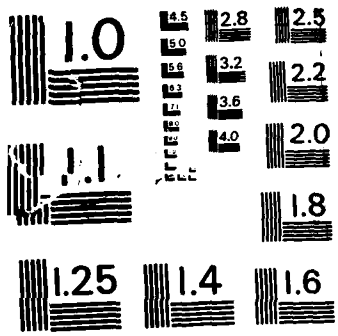
AD-A167 604 EVALUATING EXPERIMENTAL DESIGNS FOR FITTING RESPONSE SURFACES OF DETERMINISTIC MODELS(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI 1/2

UNCLASSIFIED B K ISHIHARA DEC 85 AFIT/GOR/OS/85D-10 F/G 12/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS - 1963 - A

AFIT/GOR/OS/85

(1)

EVALUATING EXPERIMENTAL DESIGNS FOR FITTING RESPONSE SURFACES OF DETERMINISTIC MODELS

THESIS

Bryan K. Ishihara
Captain, USAF

AFIT/GOR/OS/85D-10

This thesis evaluates several bias minimizing and variance minimizing experimental designs in terms of their effectiveness and efficiency in constructing response surface equations for a deterministic, nuclear exchange problem. The criteria which is used to evaluate these designs includes: 1) number of required design points; 2) number of terms in the response equation; 3) accuracy of fit of the response equation; 4) orthogonality of the design; and 5) rotatability of the design.

In addition, the response surface equations are evaluated in terms of their predictive power and their explanatory power. The predictive power addresses the equation's ability to adequately estimate the true surface and to accurately predict a future response for a given set of inputs. The explanatory power addresses the equation's ability to present a response equation which is simple to interpret so that the true surface can be easily evaluated and the results can be easily explained.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release;
Distribution Unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
AFIT/GOR/OS/85D-10

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
School of Engineering
Air Force Inst. of Technology

6b. OFFICE SYMBOL *(If applicable)*
AFIT/ENS

7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS *(City, State and ZIP Code)*
Wright-Patterson AFB, OH 45433

7b. ADDRESS *(City, State and ZIP Code)*

8a. NAME OF FUNDING/SPONSORING ORGANIZATION

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS *(City, State and ZIP Code)*

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.

11. TITLE *(Include Security Classification)*
See Box 19

12. PERSONAL AUTHOR(S)
Bryan K. Ishihara, Captain, USAF

13a. TYPE OF REPORT
MS Thesis

13b. TIME COVERED
FROM ________ TO ________

14. DATE OF REPORT *(Yr., Mo., Day)*
1985 December

15. PAGE COUNT
158

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB. GR.
12	01	

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
Response Surface Methodology, Experimental Design, Deterministic Models

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Block 11:
Title - Evaluating Experimental Designs for Fitting Response Surfaces of Deterministic Models

Advisor - Palmer W. Smith, Lt Col, USAF

Approved for public release: IAW AFR 190-1.
LYNN E. WOLAVER 13 Feb 86
Dean for Research and Professional Development
Air Force Institute of Technology (AFSC)
Wright-Patterson AFB OH 45433

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
Palmer W. Smith, Lt Col, USAF

22b. TELEPHONE NUMBER *(Include Area Code)*
(513) 255-3362

22c. OFFICE SYMBOL
AFIT/ENS

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

AFIT/GOR/OS/85D-10

EVALUATING EXPERIMENTAL DESIGNS FOR FITTING RESPONSE SURFACES OF DETERMINISTIC MODELS

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Operations Research

Bryan K. Ishihara, M.B.A., B.A., B.A.

Captain, USAF

December 1985

Approved for public release; distribution unlimited

Acknowledgements

First, I would like to thank my thesis advisor, LTC Palmer W. Smith. He provided the research topic, gave me lots of advice during this thesis, and was able to get a lot more out of me than I thought possible. He may have proved that it is possible to get blood out of a turnip.

Secondly, I would like to thank two of my classmates, Captains Kevin Smith and Ed Flowers. They automated my data collection process and saved me a lot of time on the computer. Without their help, I would never have been able to spend fifteen hours a week on the golf course this summer.

Bryan K. Ishihara

TABLE OF CONTENTS

LIST OF TABLES

Table	Page

ABSTRACT

This thesis evaluates several bias minimizing and variance minimizing experimental designs in terms of their effectiveness and efficiency in constructing response equations for a deterministic, nuclear exchnage problem. The criteria which is used to evaluate these designs includes: 1) number of required design points; 2) number of terms in the response equation; 3) accuracy of fit of the response equation; 4) orthogonality of the design; and 5) rotatability of the design.

In addition, the response surface equations are evaluated in terms of their predictive power and their explanatory power. The predictive power addresses the equation's ability to adequately estimate the true surface and to accurately predict a future response for a given set of inputs. The explanatory power addresses the equation's ability to present a response equation which is simple to interpret so that the true surface can be easily evaluated and the results can be easily explained.

I. OVERVIEW

BACKGROUND

Response Surface Methodology (RSM) is a method for fitting mathematical models to surfaces generated by experiment. RSM is concerned with the functional relationship

$$V = h(U_1, U_2, \ldots, U_N) \qquad (1)$$

between the response variable, V, and the n independent variables $U_1, U_2, \ldots, U_N$. The response relationship is usually expressed as a low order polynomial or some other non-linear function which approximates the true surface. Graphically, the response relationship is represented as a surface in a space whose coordinates are the n+1 variables, $U_1, U_2, \ldots, U_N$ and V. For example, the response surface can be expressed as a series of contours, similar to the rise and fall of land masses on a topographic map.

In applying RSM, the analyst selects an experimental design (i.e., a theory which requires the least number of points to accurately fit the model) such that the functional relationship

$$y = f(x_1, x_2, \ldots, x_N) \qquad (2)$$

fitted by the method of least squares closely approximates the true function

$$y' = g(x_1, x_2, \ldots, x_p) \qquad (3)$$

over a specified region. When f inadequately represents

g, there are two errors associated with this problem:

1) variance error - which is error due to sampling;

and 2) bias error - which is error due to the lack of fit of f to g

Initially, criteria for evaluating the adequacy of designs have focused primarily on minimizing the variance error while the question of bias error has been given somewhat secondary considerations. However, a bias minimizing design takes on added significance when RSM is applied to a deterministic model because there is no sampling error associated with this model. In this context, a deterministic model is a model in which the response variables are free from stochastic variation.

For example, non-deterministic models, such as computer simulations, yield different results (due to random sampling of distributions) each time the test is conducted. However, deterministic models, such as linear programming problems, yield the same results each time they are applied because sampling is not involved for the fixed output level. Therefore, any error associated with the deterministic model is solely attributable to bias error (lack of fit) and none of it to variance error (sampling).

In addressing the concept of experimental designs for response surfaces, most of the RSM literature has focused on those designs which minimize variance. These designs have received a large share of attention because RSM has been

almost exclusively applied to non-deterministic models. Examples of variance minimizing designs are provided in Box and Behnken's work (1:455) on three level factorial designs.

Thus far, only a limited amount of literature has been published on bias minimizing designs. Only the basic theory of bias minimizing designs exists in the literature with the applications of the theory still to be fully documented. In general, the applications of RSM have focused on generating response surface equations for two purposes: to predict future responses and to determine the input values of the optimum response. In determining the optimum operating conditions, the experimenter employs various search techniques, such as gradient search methods, to solve the problem. However, in examining the response surface of a deterministic model, the experimenter may not be interested in determining the optimum operating conditions. Rather, as in this thesis, the experimenter may be interested in adequately representing the true surface with his predicted surface so that he can explore the relationships between the input variables and accurately predict the response values. As a result, the experimenter is interested in selecting a design which allows him to employ simple analytical techniques to the response equation to examine these relationships.

The basic theory of bias minimizing designs proposes that a design can be constructed which requires fewer number of design points and which achieves a better fit of the

function to the response surface than a variance minimizing design because a bias minimizing design does not consider any variance in its formulation. Manacapilli (17:6.1) has stated that for experiments with four or more variables, a bias minimizing design provides a function with a better fit to a surface than a variance minimizing design. However, the bias minimizing design requires more design points to achieve the fit. Thus, these results indicate that not all bias minimizing designs achieve a better fit with fewer design points.

PROBLEM STATEMENT

In developing experimental designs for response surfaces, researchers have focused on variance minimizing designs as the basic criterion in order to capture the response surface. However, deterministic models possess characteristics whereby a bias minimizing design may be a more appropriate choice. Thus, the problem is to find a bias minimizing design which best fits the response surface of a deterministic model and to compare the efficiency and effectiveness of this design to a variance minimizing design.

RESEARCH QUESTION

What experimental designs produce effective and efficient response surfaces of deterministic models?

SUBSIDIARY QUESTIONS

RSM when applied to a deterministic model is

investigated to find bias minimizing experimental designs. The results of this design are evaluated over various factor levels and its effectiveness is compared to the results of variance minimizing designs. The primary measures of effectiveness (MOE) are: orthogonality of the design, accuracy of fit to the true response surface, and number of required design points.

Some of the questions to be answered from these MOEs are:

1) What are the tradeoffs of using a bias minimizing design instead of a variance minimizing design to fit the response surface of a deterministic model?

2) When is this design more efficient or more effective than a variance minimizing design?

3) Is the analysis of response surfaces made easier with a bias minimizing design?

4) Is the bias minimizing design a practical method (in terms of the required number of design points) to fit a response surface of a deterministic model?

5) Do these designs produce equivalent results?

6) Can an orthogonal bias minimizing design be constructed to facilitate the analysis of the surface?

LITERATURE REVIEW

The concepts of Response Surface Methodology were first developed by Box and Wilson in the early 1950's (5:1). This methodology is designed to be a practical and efficient manner in dealing with the problem of determining the

optimum operating conditions for a specific process. For example, RSM is ideally suited for agricultural and chemical experiments in which the experimenter seeks to maximize the yield or purity of the product. An extensive list of the applications of RSM in chemical, agricultural, and other related fields can be found in Hill and Hunter's article (11:591).

More recent applications of RSM have been in the area of animal husbandry. Articles by Toyomizu, Akiba, Horiguchi, and Matsumoto, (24:886) and by Roush, Petersen, and Arscott, (21:1504) have identified the uses of RSM in determining the optimum operating conditions for raising chickens.

With regards to military applications, Smith and Mellichamp (22) used RSM to perform multidimensional parametric analysis to study mathematical programming models. In addition, several recent Air Force Institute of Technology (AFIT) thesis efforts have applied Smith and Mellichamp's approach and techniques to various military related problems. For example, Manacapilli (17) applied RSM along with economic production functions to a nuclear exchange linear programming model. Manacapilli showed that economic production functions could be fitted to the response surface of a deterministic model so that basic economic theory could be used to analyze the surface.

Another application of RSM to a nuclear exchange model is the work by Graney (19). In his thesis, Graney combined

two different response surfaces (representing two different objectives) as a means of evaluating a multiple objective problem.

Further use of Smith and Mellichamp's paper in this research effort is defined in Chapter II under the proposed methodology section.

GENERAL METHODOLOGY

In general, the basic approach in answering the research question is to apply various experimental designs to a response surface of a deterministic model and to evaluate the differences in the results.

Specifically, the first step is to identify bias minimizing designs, such as the Box-Draper designs and the Koshal designs.

The second step is to identify variance minimizing designs. Several variance minimizing designs are considered in this study. These designs include, but are not limited to, second order rotatable designs and central composite rotatable designs. These types of designs are selected because of their frequent use in RSM.

Next, both the bias minimizing and variance minimizing designs are applied to a deterministic model. The deterministic model to be used in this research effort is a nuclear exchange model. The nuclear exchange model is a simple linear programming model designed to optimally allocate the number of strategic weapons per weapon type

(input variables) to targets so that maximum damage occurs (response variable). This model was selected because of its basic simplicity and because past AFIT thesis efforts, by Graney (19:51) and by Manacapilli (17:6.1) have used a similar model with favorable results.

Finally, the functions obtained from the two types of experimental designs are evaluated in terms of their effectiveness and efficiency.

In terms of effectiveness, the fit of the postulated surfaces to the actual surface is measured. To measure the fit of a postulated surface, a random set of data points from the actual surface is collected and is applied to the function which is used to generate the postulated surface. The differences between the expected response and the actual response are then computed. The sum of the deviations squared (SDS) is then obtained and a comparison is made between each of the functions estimated by different designs based on their SDS value. The designs which generate models which most closely represent the true surface are the model which minimizes SDS.

In terms of efficiency, the minimum number of design points required to generate the surface are evaluated. As stated earlier, Manacapilli showed that a bias minimizing design required more design points than a variance minimizing design while gaining only a small improvement in the fit of the postulated surface. If this result holds for the generalized bias minimizing design, then the

effectiveness and efficiency trade-offs between bias and variance minimizing designs are evaluated.

One of the measures of this trade-off is an effectiveness-efficiency factor. This factor is given by

$$E=(\text{error in fit of design})*(\text{number of design points}) \quad (4)$$

where the error in the fit of the design is the mean of the differences between the predicted and the actual responses divided by the actual responses.

This factor simultaneously takes into consideration the precision with which the design estimates the true surface and the number of required experimental design points (18:135).

The next chapter expands the methodology and discusses the specific concepts involved in applying RSM to a deterministic model.

II. METHODOLOGY

INTRODUCTION

This chapter details the methodology used in applying Response Surface Methodology (RSM) to a deterministic model. It builds on the general methodology described in Chapter I by explaining the various experimental designs which are used in this study, by detailing the nuclear exchange linear programming (LP) models and by outlining the application of experimental designs in generating the response surfaces.

SECOND ORDER EXPERIMENTAL DESIGNS FOR RESPONSE SURFACES

A major goal of Response Surface Methodology is to answer the question of what happens to the response variable when input variables are allowed to vary over a specified range. Instead of utilizing a one variable at a time procedure (which changes the level of one factor while fixing all other input variables at a specified level), the theory of experimental design is utilized. By applying the appropriate experimental designs, a response function is generated to estimate the response value for any given combination of input variables. Two books which are excellent references on the theory of experimental design have been written by Davies (8) and by Hicks (10). An extensive list of applications of experimental design in RSM can be found in Steinberg and Hunter's article (23:71).

Furthermore, a significant portion of RSM literature

has focused on second order experimental designs. That is, designs specifically constructed for fitting a second order polynomial equation to the data. This type of design produces a second order polynomial equation of the form :

$$y = b_0 + \sum b_i x_i + \sum b_{ii} x_i^2 + \sum b_{ij} x_i x_j \quad (5)$$

This equation includes the first few terms of a Taylor series expansion. Therefore, a second order design usually produces an adequate response equation in estimating the actual surface because the response equation depicting the actual surface can usually be represented by a Taylor series expansion. The difference between the actual and predicted surfaces is due to the higher order terms of the Taylor series expansion which are not included in the equation for the predicted surface, but are included in the equation for the actual surface.

<u>ORTHOGONALITY</u>. One of the characteristics of the experimental design which is sought by the experimenter is the mutual orthogonality of coefficients. Orthogonality is that property of an experiment which ensures that the different classes of effects shall be capable of direct and seperate estimation without any entanglement. The sum of squares of all the effects are then independent and additive (8:587).

Orthogonality permits surer assessment of those areas on which process design and control efforts should be concentrated. It also provides means for unambiguous simplification and improvement of response models along with

the potential for uncovering basic mechanisms (20:419). That is, those terms which are most important in predicting the surface can be easily identified.

ROTATABILITY. A second desirable characteristic of an experimental design is that the design be rotatable. An experimental design is said to be rotatable if the variance of the estimated response $\hat{y}$, at some point $(x_1, x_2, \ldots, x_K)$, depends on the distance from the point to the design center and not on direction. In other words, as far as the design is concerned, points in the factor space which are the same distance from the origin are treated as being equally important (18:165).

Box and Draper (3:339) indicate that a rotatable design will minimize bias provided that certain conditions are met. These conditions are: the region of interest is spherical and the fitted model is an mth order polynomial and the true model is an nth order polynomial, such that if m+n=2r, then the appropriate design is an rth order rotatable design; if m+n=2r+1, then the appropriate design is a rth order rotatable design with moments of order 2r+1 all zero. Thus, in fitting a second order polynomial to a true surface of order three, the fifth moments of the design must all be zero for the design to be rotatable.

NUMBER OF DESIGN POINTS. A third desirable characteristic of an experimental design is that the fewest possible number of design points be used to estimate the response surface. The fewest number of design points is a

function of the number of factors and the type of design being used.

The minimum possible number of design points that can be used to estimate any surface is defined by the expression $(k+1)*(k+2)/2$, where k defines the number of factors in the experiment. This quantity is in stark contrast to the number of data runs which are required if the experimenter chose to utilize a one variable at a time procedure to estimate the surface. For example, in a simple two variable problem in which the first variable is allowed to vary between 0 and 450 and the second variable is allowed to vary between 0 and 750, Manacapilli calculated that 338,701 (451 * 751) runs are required to produce the exact surface of a two factor experiment if the variables are varied one at a time (17:3.8). On the otherhand, certain experimental designs are able to estimate this surface with only six data runs. The savings in time and cost are readily apparent.

BIAS MINIMIZING DESIGNS

Since the only error associated with deterministic models is bias error, bias minimizing designs are very effective in fitting response surfaces to these models. The bias minimizing designs which are utilized in this project are: Box-Draper designs and Koshal designs.

BOX-DRAPER BIAS MINIMIZING DESIGNS. The Box-Draper bias minimizing design is constructed in a similar manner as a central composite design. That is, the design is composed of a fractional factorial portion, an axial point portion

and center points. Like the central composite design, the bias minimizing design is nearly orthogonal and nearly rotatable.

For the case of three independent variables the design matrix is given by

$$
D = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} X1 & X2 & X3 \\ \end{array}
$$

$$
D = \begin{bmatrix} \pm p & \pm p & \pm p \\ \pm q & 0 & 0 \\ 0 & \pm q & 0 \\ 0 & 0 & \pm q \\ 0 & 0 & 0 \end{bmatrix} \qquad (6)
$$

The specific values of p and q are dependent on the second and fourth moments of the design matrix. The specific values which minimize bias and a detailed derivation of the design matrix can be found in Box and Draper's article (2:622) and Myers' book (18:196).

KOSHAL DESIGNS. The Koshal design is a simple design used by Koshal in fitting frequency distributions by the method of maximum likelihood (12:577). The Koshal design requires the minimum number of design points to fit a quadratic response surface, and the major advantage of using this design is its simplicity and economy.

For the case of three independent variables the design matrix is given by

$$
D = \begin{array}{c} \\ \\ \end{array}
\begin{array}{ccc} X1 & X2 & X3 \\ \end{array}
$$

$$
D = \begin{bmatrix} -.5 & -.5 & -.5 \\ .5 & -.5 & -.5 \\ -.5 & .5 & -.5 \\ -.5 & -.5 & .5 \\ 1.5 & -.5 & -.5 \\ -.5 & 1.5 & -.5 \\ -.5 & -.5 & 1.5 \\ .5 & -.5 & .5 \\ -.5 & .5 & .5 \\ .5 & .5 & -.5 \end{bmatrix} \quad (7)
$$

A detailed derivation of the design matrix can be found in Kanemasu's article (12:578).

VARIANCE MINIMIZING DESIGNS

As stated earlier, there are two errors associated with the problem of fitting the predicted surface to the actual surface: variance error and bias error. As their names imply, variance minimizing designs are constructed to minimize variance error and bias minimizing designs are constructed to minimize bias error.

Although deterministic models do not generate any variance error, variance minimizing designs have been shown to produce adequate results in fitting response surfaces to these models. In fact, Box and Draper hypothesized that designs which minimize variance do a very good job at minimizing bias also (2:622). The variance minimizing

designs which are utilized in this project are: Central Composite Rotatable Designs, Box-Behnken Designs, Hybrid Designs, and Minimum Point Designs.

CENTRAL COMPOSITE ROTATABLE DESIGNS. The central composite rotatable design was first developed by Box and Wilson as a practical alternative to the 3^K factorial design for estimating a second order response equation. This design is probably the most widely known and recommended design for estimating quadratic response surfaces (16:412).

The central composite design is the 2^K factorial or fractional factorial, augmented by axial points and center points. For the case of three independent variables the design matrix is given by

$$D = \begin{array}{c} \begin{array}{ccc} X1 & X2 & X3 \end{array} \\ \begin{bmatrix} \pm 1 & \pm 1 & \pm 1 \\ \pm q & 0 & 0 \\ 0 & \pm q & 0 \\ 0 & 0 & \pm q \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad (8)$$

With the proper selection of q, the central composite design is orthogonal and rotatable. A detailed derivation of the q-values can be found in Box and Hunter's article (4:195).

BOX-BEHNKEN DESIGNS. In 1960, Box and Behnken developed a new class of three level factorial designs which were useful for estimating the coefficients in a second degree graduating polynomial (1:455). These designs are

nearly rotatable and nearly orthogonal (i.e., a small correlation exists between the constant term and the squared terms and between the squared terms themselves). A practical advantage of the Box-Behnken design over the full central composite design is that when evaluating problems with large number of factors (i.e., greater than five), the Box-Behnken design requires fewer design points to estimate the surface.

The designs are formed by combining factorial designs with incomplete block designs (1:457). For the case of three independent variables, the design matrix is given by

$$D = \begin{array}{c} \begin{array}{ccc} X1 & X2 & X3 \end{array} \\ \begin{bmatrix} \pm 1 & \pm 1 & 0 \\ \pm 1 & 0 & \pm 1 \\ 0 & \pm 1 & \pm 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad (9)$$

A detailed derivation of the design matrix can be found in Box and Behnken's article (1:455).

HYBRID DESIGNS. Hybrid designs were created to achieve the same degree of orthogonality as central composite designs or regular polyhedral designs, to be nearly minimum point, to be nearly rotatable, and to possess some ease in coding (20:419).

The hybrid designs are constructed in a similar manner as central composite designs. That is, the design is composed of a fractional factorial portion, an axial point portion, and center points. The major difference between

the hybrid and central composite designs is that hybrid designs are augmented with an extra variable column which resembles a cross polytope design. For the case of three independent variables the design matrix is given by

$$D = \begin{array}{ccc} X1 & X2 & X3 \\ \end{array}$$
$$D = \begin{bmatrix} \pm 1 & \pm 1 & .6386 \\ \pm 1.1736 & 0 & -.9273 \\ 0 & \pm 1.1736 & -.9273 \\ 0 & 0 & 1.2906 \\ 0 & 0 & -.1360 \\ 0 & 0 & 0 \end{bmatrix} \qquad (10)$$

A detailed derivation of the design matrix can be found in Roquemore's article (20:419).

<u>MINIMUM POINT SECOND ORDER DESIGNS</u>. The purpose of second order designs with minimum number of points is to provide a low cost, practical design to estimate a quadratic response surface. For a k-factor design, the minimum number of points is (k+1)*(k+2)/2. The design should give rise to least squares estimates with minimum generalized variance (6:613).

For the case of three independent variables, the design matrix is given by

$$
D = \begin{array}{c} \begin{array}{ccc} X1 & X2 & X3 \end{array} \\ \begin{bmatrix} -1 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \\ p & p & -1 \\ p & -1 & p \\ -1 & p & p \\ q & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & q \end{bmatrix} \end{array} \qquad (11)
$$

The specific values of p and q, along with a detailed derivation of the design matrix can be found in Box and Draper's article (6:613).

MISCELLANEOUS

Several articles have been published which have offered an alternate approach to minimizing bias error in response surfaces. The articles by: 1) Karson, Manson, and Hader (14:461), 2) Karson (13:1565), 3) Karson and Spruill (15:329), and 4) Cote, Manson and Hader (7:633), have applied similar techniques to address the problem of minimizing bias. That is, their methods consist of developing an estimator for a given design which minimizes bias.

Although these works will not be used in this project, their availability is noted here for any future work on this topic.

NUCLEAR EXCHANGE MODEL

The deterministic model used to generate the response values for this study is a nuclear exchange linear programming model. In this rather simplistic model of a nuclear force structure, the single objective of the model is to maximize the damage expectancy of the force structure. The damage expectancy is maximized given the effectiveness of each weapon type against each target type.

For example, consider the hypothetical strategic force of three weapon systems (ICBM, Bomber, SLBM) and two target types. Table 1 shows the amount of damage expected (D1, D2, D3,..., D6) given a particular weapon type and target type.

TABLE 1

SINGLE SHOT DAMAGE EXPECTANCY TABLE

	WEAPON TYPE (FACTORS)		
TGT TYPE	ICBM (M1)	BOMBER (B1)	SLBM (S1)
TARGET 1	D1	D2	D3
TARGET 2	D4	D5	D6

The damage expectancy values in Table 1 represent single shot damage expectancv, that is, the damage expectancy from a single warhead of a particular weapon against a single target. In this model, an additional warhead could be launched at the same target. Therefore, it is possible to increase the damage expectancy against a single target by

launching two warheads (of the same or different weapon type) at the target. In those cases where two warheads are launched at the same target, the new damage expectancy value for that target can be computed in the following manner

$$DE_{NEW} = 1 - [(1-DE_1)*(1-DE_2)] \qquad (12)$$

Thus, there are six additional combinations of damage expectancy which must be considered for each target type (e.g., M1*M1, M1*B1, M1*S1, B1*B1, B1*S1, S1*S1). After considering all possible combinations, the LP model now maximizes the total damage expectancy subject to the limitations on the number of weapons, number of targets, and other targeting constraints. Further details of the specific model used to generate the data along with the LP formulation are available in Appendix A.

APPLICATION OF THE EXPERIMENTAL DESIGN

As mentioned earlier, there is a practical advantage in employing the theory of experimental design in estimating a response surface. For example, in the simple three factor problem used in this study, if each factor were allowed to vary one at a time, the total number of runs required to generate the surface is 14,601,339,651 (1351*1801*6001). By constructing the appropriate experimental design, the surface could be estimated with as few as ten runs.

The experimental designs which are used in this study are second order designs. The purpose of a second order design is to fit a second order response equation to the

data. The second order response equation for the three factor case is of the form

$$
\begin{aligned}
DE = b_0 &+ b_1 (M1) + b_2 (B1) + b_3 (S1) \\
&+ b_{11} (M1*M1) + b_{22} (B1*B1) + b_{33} (S1*S1) \\
&+ b_{12} (M1*B1) + b_{13} (M1*S1) + b_{23} (B1*S1) \qquad (13)
\end{aligned}
$$

where, M1 = number of ICBM warheads, B1 = number of bomber warheads, and S1 = number of SLBM warheads.

One of the requirements to produce a second order design is that at least three levels of each factor must be selected so that the coefficients of the response equation can be estimated. The three levels which are usually selected are the values at the upper end (having a coded value of +1) and lower end (having a coded value of -1) of the range for each input parameter plus the mid-point (having a coded value of 0) between the extreme points. These points are selected and coded in this manner so that the coefficients of the response equation are not highly correlated.

Table 2 indicates the uncoded and coded values of the three factor problem used in this project.

TABLE 2

FACTOR LEVELS

WEAPON TYPES (FACTORS)	LEVELS (CODED VALUES)		
M1	1650(-1)	2325(0)	3000(+1)
B1	1200(-1)	2100(0)	3000(+1)
S1	6000(-1)	9000(0)	12000(+1)

In many of the central composite designs which are used in this study, the coded values are greater than +1 and less than -1. The appearance of these values can best be explained by examining the central composite design for the three factor case. In this situation, the central composite design selects its initial design points (factorial portion) at the corners of a cube centered at the origin. Because the region of interest is taken to be spherical, the design positions additional points along the axes so that representative points can be sampled over the entire region. These axial design points lie near the boundary of the sphere and along one of the axes. Therefore, the axial points have values greater than +1 and less than -1 due to the radius of the sphere.

There is a possibility of significant error in the estimated response values because the axial points are positioned outside the original feasible region. That is, if the true surface is radically different outside the feasible region than it is within the feasible region, a significant error occurs due to the positioning of the

axial points. In that case, a different means of coding or a different design (which does not utilize points outside this region) should be employed. However, if the true surface is not radically different outside the feasible region than it is within the feasible region, then the axial design points do not contribute a significant error in estimating the predicted surface.

Furthermore, many of the designs which have been selected in this study have been originally constructed with multiple center points. Often times, multiple center points are used to minimize variance, to achieve rotatability or to achieve orthogonality. However, the replication of the center points is not required while analyzing deterministic models because there is no sampling error associated with this type of model. Although pure rotatability and orthogonality may be sacrificed in these cases, the designs still maintain a high degree of rotatability and orthogonality. Therefore, only one center point is included in any design which has been originally constructed with multiple center points.

The designs which are used in this study for the three factor case have already been outlined. The designs which are used in the three, four, five, and six factor problems are provided in Appendix B.

After selecting the appropriate experimental design and after defining the LP, the next step in the methodology is to execute the appropriate number of force allocation runs

using the LP model. One run is accomplished for each of the identified combinations in order to obtain the optimum damage expectancy for each set of input factor values.

The final steps in obtaining a response equation is to use the results of a particular design as input to a multiple linear regression program. The coded levels of each of the factors are input to the program as the independent variables, and the damage expectancy is input as the dependent variable. Using multiple stepwise linear regression, the coefficients of the response equation are determined. The coded values are then uncoded in order to produce a relationship in which the actual number of warheads of the weapon systems can be used. The response equation for uncoded values portrays the relationship of each factor to the response variable. A sample of the input and the output for the stepwise linear regression is found in Appendix C.

The next chapter portrays the results and details the analysis of the effectiveness and efficiency of the various designs used in this study.

III. RESULTS AND ANALYSIS

INTRODUCTION

This chapter presents the analysis of the response equations for the various designs which are used in this study. The purpose of this analysis is not to evaluate the LP but rather, to evaluate the designs and the response equations.

The analysis is divided into two parts: the predictive power of the response equations and the explanatory power of the response equations. The predictive power of the response equation addresses the equation's ability to adequately estimate the true surface and to accurately predict a response for a given set of inputs. The explanatory power addresses the equation's ability to present a response equation which is simple to interpret so that the true surface is easily evaluated and the results are easily explained.

PREDICTIVE POWER OF RESPONSE SURFACE EQUATIONS

As stated in Chapter I, the basis for evaluating the predictive capabilites of an equation of a response surface design is eq(4), the effectiveness - efficiency factor.

$$E=(\text{error in fit of design})*(\text{number of design points}) \quad (4)$$

This factor generates a value so that various designs are compared in terms of their precision in estimating the

true surface and the number of required experimental design points.

In order to obtain the error in fit of the design, a random set of points is used as inputs into the response surface equation and the differences between the actual and predicted responses are recorded. In comparing these response equations, the random data set is not used to estimate any response equation, but rather to compare the predicted values of the estimated equations against each other. The benefit of the random data set is that all equations are compared against a common data set so that prediction bias is avoided. Prediction bias arises when the choice of a regression equation is uniquely related to the observations from which it was created.

The response equations which are used to evaluate the error in fit of the design are modified versions of the full regression equations. That is, selected terms from the full regression equations have been omitted from the modified regression equaitons because these terms account for less than two percent of the total sum of squares.

The primary reason for excluding these terms is to obtain a relatively simple equation (i.e., an equation with as few terms as possible) which is easy to interpret, but still produces a good fit. If the full regression equation is used instead of the modified equation to estimate the true surface, the ability of the response equation to predict future response values is improved; however, the ability to interpret the equation is significantly

degraded. That is, the interpretation of the equation is very difficult due to the complexity of the equation. Thus, by selecting two percent of the total sum of squares as the criteria for determining what terms to include in the response equation, a reasonably good fit is obtained while still preserving the interpretative value of the response equation.

The response surface equations along with the random data sets which are used in the analysis of the predictive power can be found in Appendix D.

Tables 3 through 6 depict the results of the predictive power of the various designs which are used in this study.

TABLE 3.

PREDICTIVE POWER OF THREE FACTOR DESIGNS

DESIGN		PERCENT ERROR IN FIT TO RANDOM DATA SET	NUMBER OF DESIGN POINTS	E - VALUE
VARIANCE DESIGNS				
BOX-BEHNKEN	*	1.08	13	14.04
HYBRID 310+CP	*	1.28	11	14.08
HYBRID 311A	*	1.63	11	17.93
CENTRAL COMP	*	1.56	15	23.40
MIN POINT		6.12	10	61.20
BIAS DESIGNS				
BOX-DRAPER	*	1.38	15	20.70
KOSHAL		2.74	10	27.40

* indicates nearly rotatable design

TABLE 4.

PREDICTIVE POWER OF FOUR FACTOR DESIGNS

DESIGN		PERCENT ERROR IN FIT TO RANDOM DATA SET	NUMBER OF DESIGN POINTS	E - VALUE
VARIANCE DESIGNS				
HYBRID 416A+CP	*	1.06	17	18.02
HYBRID 416C	*	1.80	16	28.80
BOX-BEHNKEN	*	1.18	25	29.50
CENTRAL COMP	*	1.31	25	32.75
MIN POINT		4.37	15	65.55
BIAS DESIGNS				
BOX-DRAPER	*	0.96	25	24.00
KOSHAL		1.80	15	27.00

* indicates nearly rotatable design

TABLE 5.

PREDICTIVE POWER OF FIVE FACTOR DESIGNS

DESIGN		PERCENT ERROR IN FIT TO RANDOM DATA SET	NUMBER OF DESIGN POINTS	E - VALUE
VARIANCE DESIGNS				
CENTRAL COMP (HALF REPLICATE)	*	1.73	27	46.71
BOX-BEHNKEN	*	1.41	41	57.81
MIN POINT		5.88	21	123.48
BIAS DESIGNS				
KOSHAL		1.15	21	24.15
BOX-DRAPER (HALF-REPLICATE)	*	1.37	27	36.99
BOX-DRAPER (FULL)	*	1.31	43	56.33

* indicates nearly rotatable design

TABLE 6.

PREDICTIVE POWER OF SIX FACTOR DESIGNS

DESIGN		PERCENT ERROR IN FIT TO RANDOM DATA SET	NUMBER OF DESIGN POINTS	E - VALUE
VARIANCE DESIGNS				
HYBRID 628A	*	1.33	28	37.24
HYBRID 628B	*	1.35	28	37.80
MIN POINT		1.76	28	49.28
BOX-BEHNKEN	*	1.15	49	56.35
CENTRAL COMP (HALF-REPLICATE)	*	1.33	45	59.85
BIAS DESIGNS				
KOSHAL		1.72	28	48.16
BOX-DRAPER (HALF-REPLICATE)	*	1.39	45	62.55

* indicates nearly rotatable design

ACCURACY. Tables 3 through 7 indicate that the accuracy of a response surface equation is dependent on the number of design points and on the degree of rotatability of the design. That is, the accuracy of a response surface equation is highly correlated to the number of design points, but this relationship is not a perfect correlation and their differences in fit may be explained by the degree of the rotatability of the designs.

For example, it may be hypothesized that the design

with the most points produces the most accurate fit. That is, each design selects its design points under the assumption that the design points are representative of their surrounding area. In theory, by selecting more design points, the area for which each point is an estimate is decreased. Thus, the total error in the estimate of the true surface is reduced. But as Tables 3 through 7 indicate, more design points does not guarantee a more accurate fit. This result is rather surprising, but it does indicate that other contributing factors influence the degree of accuracy of the response equation other than the number of design points.

A major contributing factor which influences the accuracy of an equation of a response surface design may be the degree of rotatability of the design. In general, the variance and bias minimizing designs are able to generate similar degrees of fit in estimating a response surface equation to the true surface, provided that the variance minimizing designs are nearly rotatable. Rotatability is an important characteristic for variance minimizing designs when estimating deterministic models because the selection of the design points is not strictly based on minimizing bias. But, as stated in Chapter I, if the region of interest is spherical, rotatable designs minimize bias. The importance of rotatability in minimizing bias is highlighted by the fact that the only variance minimizing design which fails to consistently produce a relatively accurate fit is

the minimum point design, which is a non-rotatable design.

However, none of the designs mentioned thus far is perfectly rotatable; thus, bias is not totally minimized for these designs. In fact, all of the designs, except the minimum point and the Koshal designs, are nearly rotatable. That is, these nearly rotatable designs have varying degrees of rotatability and thus, have varying degrees of effectiveness in minimzing bias. Therefore, it might be hypothesized that the differences in accuracy between the designs which are not accounted for by the number of design points, are accounted for by the degrees of rotatability of the designs.

For example, the response surface equation of the hybrid design (416A+CP) achieves a more accurate fit to the true surface than the response equation of the Box-Behnken design for the four factor problem. In this example, the hybrid design requires eight fewer points than the Box-Behnken design, but is able to achieve a higher degree of accuracy. By the hypothesis, this indicates that the hybrid design (416A+CP) has a higher degree of rotatability than the Box-Behnken design for four factors. This hypothesis is confirmed by the fact that the Box-Behnken design is not highly rotatable for four factors, while the hybrid design maintains a high degree of rotatability.

A similar argument can be made to explain why no single design is able to achieve the most accurate fit for all factors. That is, for the different number of factors,

these designs have different number of design points and different degrees of rotatability. Thus, the degrees of accuracy for the response equations are affected.

EFFECTIVENESS-EFFICIENCY FACTOR. Since the accuracy of fit of the design is highly correlated to the number of design points, the results which are obtained concerning the effectivenss-efficiency factor are not surprising. That is, a design which provides the most accurate fit with the fewest number of design points does not exist. Even a perfectly rotatable design does not provide the most accurate fit, if a sufficient number of design points are not specified.

Thus, in attempting to minimize the effectiveness-efficiency factor, it remains for the experimenter to evaluate the problem to determine what degree of accuracy is required and how many design points are available. In evaluating the trade-off between accuracy and the number of design points, one major consideration is the cost of performing additional data runs. In those problems in which cost of executing additional data runs is minimal, the experimenter may concentrate on achieving a higher degree of accuracy. On the other hand, in those cases in which set-up costs and run time are prohibitive, the experimenter may select the most economical design (in terms of the fewest design points) which achieves an adequate level of accuracy.

The trade-offs between accuracy (level of fit) and cost (number of design points) are further complicated when the

experimenter desires a response equation which has superior explanatory powers.

EXPLANATORY POWER OF RESPONSE SURFACE EQUATION

The basis for evaluating the explanatory capabilities of an equation of a response surface design is the interpretation of the coefficients of the response equation. The coefficients provide answers to such questions as:

> "What is the overall effect to the dependent variable, if one independent variable is increased by an amount X and all other independent variables are held constant?"

By applying the analysis techniques of multiple regression, similar "what...if..." questions can be answered quickly and accurately. The answers to these questions are quick and easy because the basic interpretation of the coefficients in multiple regression analysis is that the coefficients of the main effect terms represent the marginal effects of change of the dependent variable when the values of the independent variables are altered.

EFFECT OF MULTICOLLINEARITY ON REGRESSION COEFFICIENTS. As stated earlier, Box and Hunter have identified a small correlation that exists between the constant term and the squared term and between the squared terms themselves (4:201). This correlation is commonly referred to as multicollinearity in statistical literature. Since Box and Hunter concentrated on the predictive power of the response equation, multicollinearity did not affect

their results because of the fact that when some or all independent variables are correlated among themselves, the ability to obtain a good fit is not generally inhibited nor does it tend to affect prediction of new observations (19:384). That is, the predictive power of the response equation is not generally affected when multicollinearity exists.

However, the common interpretation of regression coefficients as measuring the change in the expected value of the dependent variable when the corresponding independent variable is increased by one unit while all independent variables are held constant is not fully applicable when a high degree of multicollinearity exists. While it may be conceptually possible to vary one independent variable and hold the others constant, it may not be possible in practice to do so for independent variables that are highly correlated (19:385). Because a high degree of correlation among the independent variables exists in non-orthogonal designs, these designs do not possess a high degree of explanatory power by the fact that the regression coefficients of the response equations do not accurately measure the marginal effects of change of the dependent variable when the values of the independent variables are altered. Thus, only those designs which have a high degree of orthogonality will be considered in the explanatory power section of this study. The designs which are to be evaluated are: the Central Composite Rotatable designs, the

Box-Behnken designs, the Hybrid designs, and the Box-Draper Bias Minimizing designs.

THREE FACTOR RESULTS. Equation (14) is the basic response equation for the three factor problem.

$$DE = b_0 + b_1(M1) + b_2(B1) + b_3(S1) - b_{33}(S1*S1) - b_{23}(B1*S1) \quad (14)$$

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 7 lists the coefficients of the response equation for the various designs for the three factor problem.

TABLE 7.

COEFFICIENTS OF THE RESPONSE EQUATION
FOR THREE FACTOR DESIGNS

DESIGN	COEFFICIENTS					
	b_0	b_1	b_2	b_3	b_{33}	b_{23}
(1) Central Comp	124.9285	.4848	1.153	1.004	.0000276	.000062
(2) Box-Behnken	-648.3319	.4965	1.225	1.180	.0000375	.0000665
(3) Hybrid (311A)	68.4118	.5243	1.195	.961	.0000251	.0000602
(4) Hybrid (310+CP)	-626.1627	.5002	1.327	1.163	.0000352	.0000774
(5) Box-Draper	-285.6179	.4798	1.223	1.085	.0000318	.0000667

<u>FOUR FACTOR RESULTS</u>. Equation (15) is the basic response equation for the four factor problem.

$$DE = b_0 + b_1(M1) + b_2(M2) + b_3(B1) + b_4(S1) - b_{44}(S1*S1) - b_{24}(M2*S1) - b_{34}(B1*S1) \qquad (15)$$

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 8 lists the coefficients of the response equation for the various designs for the four factor problem.

TABLE 8.

COEFFICIENTS OF THE RESPONSE EQUATION
FOR FOUR FACTOR DESIGNS

DESIGN	COEFFICIENTS							
	b_0	b_1	b_2	b_3	b_4	b_{44}	b_{24}	b_{34}
(1) Central Comp	939.3	.416	.961	1.02	.907	.000023	.000054	.000051
(2) Box-Behnken	1320.0	.383	.946	.987	.877	.000023	.000054	.000051
(3) Hybrid (416C)	1129.1	.434	1.12	1.08	.793	.000015	.000068	.0000645
(4) Hybrid (416A+CP)	1060.1	.439	1.09	1.11	.823	.000017	.000064	.0000601
(5) Box-Draper	1332.3	.395	.944	.988	.887	.000024	.000053	.0000503

FIVE FACTOR RESULTS. Equation (16) is the basic response equation for the five factor problem.

$$DE = b_0 + b_1(M1) + b_2(M2) + b_3(B1) + b_4(B2) + b_5(S1) - b_{55}(S1*S1) \qquad (16)$$

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 9 lists the coefficients of the response equation for the various designs for the five factor problem.

TABLE 9.

COEFFICIENTS OF THE RESPONSE EQUATION
FOR FIVE FACTOR DESIGNS

DESIGN	COEFFICIENTS						
	b_0	b_1	b_2	b_3	b_4	b_5	b_{55}
(1) Central Comp	1615.7	.501	.630	.712	.755	.951	.0000261
(2) Box-Behnken	844.0	.603	.680	.729	.769	1.04	.0000295
(3) Box-Draper (H)	536.6	.604	.683	.739	.785	1.09	.0000329
(4) Box-Draper (F)	388.0	.629	.678	.756	.798	1.09	.0000329

SIX FACTOR RESULTS. Equation (17) is the basic response equation for the six factor problem.

$$DE = b_0 + b_1(M1) + b_2(M2) + b_3(M3) + b_4(B1) + b_5(B2) + b_6(S1) - b_{66}(S1*S1) - b_{56}(B2*S1) \quad (17)$$

The terms which have been omitted from the equation account for less than two percent of the total sum of squares, and therefore are dropped from the final equation.

Table 10 lists the coefficients of the response equation for the various designs for the six factor problem.

TABLE 10.

COEFFICIENTS OF THE RESPONSE EQUATION FOR SIX FACTOR DESIGNS

DESIGN	COEFFICIENTS								
	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_{66}	b_{56}
(1) Central Composite	620.1	.562	.606	.673	.685	1.24	1.05	.0000235	.000057
(2) Box-Behnken	-642.1	.536	.628	.654	.692	1.39	1.28	.0000331	.000073
(3) Hybrid (628A)	715.9	.563	.598	.626	.684	1.23	1.04	.0000228	.000057
(4) Hybrid (628B)	695.9	.554	.606	.614	.683	1.23	1.04	.0000232	.000057
(5) Box-Draper	-101.0	.568	.645	.678	.695	1.22	1.21	.0000328	.000054

INTERPRETATION OF COEFFICIENTS. The parameter b_0 is the y-intercept of the regression surface. This term does not have any particular meaning as a separate term in any of the regression models because the range of the variables in the model does not include the point where M1=0,...,S1=0. If this point had been in the relevant range of the variables, the b_0 term would indicate the response level when all of the independent variables are set to zero.

When the regression model does not include cross product or squared terms (i.e., a first order equation), the parameters b_K ($k \neq 0$) indicate the change in the mean response per unit increase in x_K when all other variables included in the model are held constant. That is, b_1 indicates the change in the expected value of the response with a unit increase in x_1 when all other variables in the model are held constant. Since the experimental designs which are being considered in this section are nearly orthogonal, the independent variables are almost uncorrelated and the variables have an additive effect.

When the regression model includes cross product or squared terms, such as Eq (14) of the three factor model, the coefficients of the main effect terms no longer indicate the change in the mean response per unit increase in x_K for any given level of the remaining variables, if that variable is also included in the cross product or squared terms. For example, in Eq(14), b_2 no longer indicates the change in the

mean response for a unit increase in B1 because of the interaction of B1 and S1. Similarly, b_3 no longer indicates the change in the mean response for a unit increase in S1 because of the interaction of S1 and B1 and because of the interaction of S1 with itself. Because of the complexity of a regression model which includes several cross product and squared terms, the most desirable regression equation which explains the response surface is one in which the main effects dominate and the cross product and squared terms terms take minor, secondary roles. In this manner, the parameters b_K would indicate the change in the mean response per unit increase in x_K when all other variables included in the model are held constant.

<u>Differences between Coefficients</u>. Tables 7 through 10 indicate that no two equations have the same coefficients to estimate the same surface. That is, each design produces a unique response equation to estimate the true surface. These results are not unexpected and does not disqualify the use of multiple regression analysis techniques as an analytical tool when evaluating response surfaces of deterministic models.

For example, small differences between the coefficients are to be expected because the predicted surface is not a perfect representation of the actual surface. The errors in the predicted surface are due to two factors: first, each design selects different quantities of sample points to estimate the true surface and each design selects its design

points at different locations in the region of interest; and second, a small amount of multicollinearity is still present between some of the variables in these equations.

The selection of the number of design points and the placement of the design points have a tremendous impact on the fitted regression function because the response equation is influenced by whatever design points are selected. Because each of the designs selects points from different areas of the true surface, the response equation must necessarily be different for each design. Any change in the selection of the design points influences the coefficients of the response equation to accomodate this change.

For example, the response equation of the hybrid design (311A) is

$$y = .5243\ (M1) + 1.195\ (B1) + .961\ (S1) - .00000251\ (S1*S1) - .0000602\ (B1*S1) + 68.4118 \quad (18)$$

If the design is altered so that the first and third variables in the design are switched, then the new design matrix is coded as follows: the S1 variable is coded as variable 1; the B1 variable is still coded as variable 2; and the M1 variable is coded as variable 3. As a result, different response values are obtained from the same design matrix because different uncoded values have been used in the MPOS input files. The resulting response equation for

the altered hybrid design is:

$$y = .4678\ (M1) + 1.11095\ (B1) + .98925\ (S1) - .0000275\ (S1*S1) - .0000512\ (B1*S1) + 209.2299 \quad (19)$$

The predictive power for both the original and the altered designs are equivalent. That is, the degrees of accuracy for both equations are the same. Howe er, a totally different set of coefficients has been generated from the altered design to estimate the true surface.

A second reason to account for the small differences between the coefficients is the small amount of multicollinearity which exists between the intercept term and the squared terms and between the squared terms themselves. Variables which possess even a small amount of multicollinearity, produce regression coefficients which tend to vary from one design to the next.

For example, a regression equation is developed from a representative sample of points (design 1) from a large population. If multicollinearity exists between some or all of the variables, a second sampling (design 2) from the population will necessarily produce a different regression equation because of the multicollinearity between the variables. The degree of variability between the coefficients of the two equations may be directly related to the degree of multicollinearity between the variables.

Multicollinearity can be eliminated between the variables of an experimental design, if the design is made

perfectly orthogonal. A full factorial design is perfectly orthogonal, if the required transformations are performed on the squared terms of the design. (The required transformation is: $X_i^2 - \bar{X}_i^2$.) The full factorial design is not commonly used in RSM because it requires a large number of design points. However, this design serves as an excellent standard to which other designs can be compared.

By performing the necessary transformations, the full factorial three factor design for the determinstic model produces the following response equation:

$$
\begin{aligned}
y = {} & .4979\ (M1) + 1.216\ (B1) + 1.1978\ (S1) \\
& - .00003865\ (S1*S1) - .0000649\ (B1*S1) \\
& - 964.8424
\end{aligned}
\qquad (20)
$$

The response equation of the full factorial design achieves the same degree of accuracy as the other nearly orthogonal designs in estimating the true surface. Thus, the primary difference between the full factorial design and the other nearly orthogonal designs is that multicollinearity does not exists between the variables in the full factorial design so that all of the variables are uncorrelated.

In that case, the full factorial design provides the most representative estimate of the true surface. That is, the regression equation which is developed from the full factorial design is strictly determined by the number and placement of its design points. Whereas, the response equations of the other designs are also determined by the

number and placement of the design points, but, the effects of multicollinearity also influence the coefficients of some of the variables of the response equations.

Thus, the degree of variability which exists between the coefficients of the full factorial response equation and the coefficients of the other response equations may be a measure of the degree of multicollinearity which exists in those designs. For example, the coefficients of the response equation for the Box-Behnken design compare more favorably to the full factorial design than some of the other three factor designs. This may indicate that the influence of multicollinearity for three factor designs, is not as significant on the variables of the Box-Behnken design as on the variables of the other designs. However, this statement is just a hypothesis because there is no measure of the degree of orthogonality for the various designs.

In short, the effects of multicollinearity on response surface equations is of primary importance when evaluating deterministic models because the analyst is attempting to draw direct inferences from the coefficients of the response equation. If the degree of multicollinearity is too great between the variables in the equation, any information obtained from the coefficients may be misleading.

As stated earlier, a small amount of multicollinearity exists in all of the equations (except the full factorial), therefore, no two designs produce the same set of

coefficients when estimating a surface. Although the differences between the coefficients are small, their effects may be quite significant.

To further address the subject matter of the effects of the differences between the coefficients, the designs and the response equations of the five factor model are evaluated. The five factor model is selected because of the simplicity of the response equations. These equations include five main effect terms and one squared term in its modified form, eq(16). In this manner, the coefficients of the M1, M2, B1 and B2, terms should represent the marginal effects of change of the dependent variable when the independent variables are altered. The coefficient of the S1 term is excluded from the list because the effects of change caused by S1 are influenced by the S1-squared term. Therefore, the marginal effects of S1 vary for different levels of S1. Tables 11 through 14 compare the predicted change to the actual change of the dependent variable when one independent variable is altered while all other independent variables are held constant.

TABLE 11.

ACTUAL AND PREDICTED CHANGES
WHEN M1 IS DECREASED BY 100

DESIGN	CHANGE IN RESPONSE ACTUAL	PREDICTED	PERCENT ERROR
(1) Central Composite	-57.0	-50.1	12.1
(2) Box-Behnken	-57.0	-60.3	5.8
(3) Box-Draper (H)	-57.0	-60.4	6.0
(4) Box-Draper (F)	-57.0	-62.9	10.4

TABLE 12.

ACTUAL AND PREDICTED CHANGES
WHEN M2 IS INCREASED BY 400

DESIGN	CHANGE IN RESPONSE ACTUAL	PREDICTED	PERCENT ERROR
(1) Central Composite	260.0	251.9	3.1
(2) Box-Behnken	260.0	271.8	4.5
(3) Box-Draper (H)	260.0	273.0	5.0
(4) Box-Draper (F)	260.0	271.0	4.2

TABLE 13.

ACTUAL AND PREDICTED CHANGES
WHEN B1 IS INCREASED BY 200

DESIGN	ACTUAL	PREDICTED	PERCENT ERROR
	CHANGE IN RESPONSE		
(1) Central Composite	140.0	142.5	1.8
(2) Box-Behnken	140.0	145.9	4.2
(3) Box-Draper (H)	140.0	147.9	5.6
(4) Box-Draper (F)	140.0	151.1	7.9

TABLE 14.

ACTUAL AND PREDICTED CHANGES
WHEN B2 IS DECREASED BY 500

DESIGN	ACTUAL	PREDICTED	PERCENT ERROR
	CHANGE IN RESPONSE		
(1) Central Composite	-370.0	-377.25	2.0
(2) Box-Behnken	-370.0	-384.5	3.9
(3) Box-Draper (H)	-370.0	-392.3	6.0
(4) Box-Draper (F)	-370.0	-398.8	7.8

There are two primary reasons to explain the discrepancies between the actual and the predicted changes of the various response surface equations.

The first reason for these discrepancies is due to multicollinearity. As was stated earlier, the effects of multicollinearity influence the coefficients of the correlated variables, and thereby influence their ability to accurately measure the marginal effects of change.

The second reason for these discrepancies is due to the fact that several terms have been omitted from the full regression model. In this five factor example, ten interaction terms and four squared terms have been removed. If these variables are inserted into eq(16), the differences between the predicted change and the actual change of the dependent variable is reduced. The full regression equation for the five factor model is

$$\begin{aligned} y = {} & b_0 + b_1(M1) + b_2(M2) + b_3(B1) + b_4(B2) + b_5(S1) \\ & + b_{12}(M1*M2) + b_{13}(M1*B1) + b_{14}(M1*B2) + b_{15}(M1*S2) \\ & + b_{23}(M2*B1) + b_{24}(M2*B2) + b_{25}(M2*S1) + b_{34}(B1*B2) \\ & + b_{35}(B1*S1) + b_{45}(B2*S1) + b_{11}(M1*M1) + b_{22}(M2*M2) \\ & + b_{33}(B1*B1) + b_{44}(B2*B2) + b_{55}(S1*S1) \end{aligned} \tag{21}$$

Tables 15 through 18 contain the same information as Tables 11 through 14, except for the fact that the full regression model (5 main effects, 10 interaction terms, 5 squared terms, and 1 intercept term) is used to determine

the predicted change of the response values in Tables 15 through 18.

TABLE 15.

ACTUAL AND PREDICTED CHANGES WHEN M1 IS DECREASED BY 100 (FULL MODEL)

	CHANGE IN RESPONSE		
DESIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1) Central Composite	-57.0	-57.5	0.9
(2) Box-Behnken	-57.0	-59.9	5.1
(3) Box-Draper (H)	-57.0	-60.2	5.6
(4) Box-Draper (F)	-57.0	-60.3	5.8

TABLE 16.

ACTUAL AND PREDICTED CHANGES WHEN M2 IS INCREASED BY 400 (FULL MODEL)

	CHANGE IN RESPONSE		
DESIGN	ACTUAL	PREDICTED	PERCENT ERROR
(1) Central Composite	260.0	247.6	4.8
(2) Box-Behnken	260.0	267.5	2.9
(3) Box-Draper (H)	260.0	272.5	4.8
(4) Box-Draper (F)	260.0	269.8	3.8

TABLE 17.

ACTUAL AND PREDICTED CHANGES WHEN B1
IS INCREASED BY 200 (FULL MODEL)

DESIGN	ACTUAL	PREDICTED	PERCENT ERROR
	CHANGE IN RESPONSE		
(1) Central Composite	140.0	140.1	0.1
(2) Box-Behnken	140.0	145.1	3.6
(3) Box-Draper (H)	140.0	146.9	4.9
(4) Box-Draper (F)	140.0	148.2	5.8

TABLE 18.

ACTUAL AND PREDICTED CHANGES WHEN B2
IS DECREASED BY 500 (FULL MODEL)

DESIGN	ACTUAL	PREDICTED	PERCENT ERROR
	CHANGE IN RESPONSE		
(1) Central Composite	-370.0	-364.1	1.6
(2) Box-Behnken	-370.0	-377.9	2.1
(3) Box-Draper (H)	-370.0	-387.1	4.6
(4) Box-Draper (F)	-370.0	-389.5	5.3

From the full regression models, the analyst is able to obtain very effective measures of the marginal effects of

the independent variables. The procedure to obtain this measure consists of calculating the response value from the response equation twice. During the first calculation, all variables are set to a base level; during the second calculation, all variables remain fixed, except for the variable whose marginal effect is to be measured. Subsequently, the difference in the response values of the two calculations represents the marginal effect of that variable.

If the full regression model is used to measure the marginal effects of an independent variable, then the requirement for low multicollinearity between the variables is eliminated. The analyst does not obtain the marginal effects information from the coefficients, but from the difference between the estimated response values. Since multicollinearity does not affect a design's ability to produce a response equation that predicts effectively, the response equation of a non-orthogonal design is able to measure the marginal effects of an independent variable as well as the equation of an orthogonal design.

The drawback to using the full regression model is the complexity of eq(21). The ability to measure the marginal effects is no longer a simple proposal and the marginal effects of an independent variable is dependent on the level at which it is evaluated. Essentially, the analyst performs a one variable at a time procedure to obtain his information. Therefore, the method involving the modified

regression model may be a more attractive alternative because of its simplicity, provided that the decision maker is able to accept some limitations in the magnitudes of the estimates of the predicted marginal effects.

<u>Analytical Value of the Modified Response Equations</u>. The question of whether or not any error level is too great is dependent on the degree of accuracy which the decision maker requires. An important point which the decision maker must consider, is that the information pertaining to the marginal effects of the independent variables is readily available from the simplified response equation, but not easily obtainable from other sources.

In examiming the predicted marginal effects of the modified equations, Tables 11 through 14 indicate that there are discrepancies between the predicted and the actual changes in the response values. The range of the error between the predicted and actual change is 1.8% to 12.1%. In general, an error of less than 2% is likely to be accepted in just about any circumstance. However, an error of 12% may not be acceptable in certain situations.

A simple test to check the magnitude of the error of the predicted change for the coefficients of the modified response equation, is to compare the predicted change of the modified response equation to the predicted change of the full regression equation. If the level of error is acceptable to the decision maker, the modified response equation is used and is a very useful analytical tool.

However, if the level of error is not acceptable, the analyst may choose to use the full regression equation to measure the marginal effects of change for the independent variables. But as was noted earlier, this procedure may be rather lengthy, in that the marginal effects of change of any variable is not constant throughout the entire surface. Therefore, the variables must be evaluated at several different levels in order to determine the marginal effect.

An alternative to using the full regression model when the level of error of the modified equation is unacceptable, is to evaluate the ratios of the coefficients of the modified equations. In regression analysis, the ratio of the coefficients indicates the contribution of one factor as compared to another factor. Therefore, although the exact magnitude of their impact on the response value can not be determined, the relative importance of the various independent variables can be assessed by rank ordering the ratios of the coefficients.

For example, the modified response equation for the five factor Box-Behnken design is

$$\begin{aligned} y = {} & .6029\ (M1) + .6795\ (M2) + .7294\ (B1) \\ & + .7689\ (B2) + 1.0363\ (S1) \\ & - .0000295\ (S1*S1) + 843.9903 \end{aligned} \tag{22}$$

The mean contribution of B2 to the response value, y, is estimated to be 1.27 times as great as the mean contribution

of M1 to the response value. This amount is obtained by calculating the ratio of the coefficients for the two independent variables (.7689/.6029).

In a similar manner, eq(23) is the response equation for the five factor Central Composite Rotatable design. According to this response equation, the mean contribution of B2 is 1.5 times as great as the mean contribution of M1 to the response value (.7545/.5011).

$$\begin{aligned} y' = {} & .5011\ (M1) + .6289\ (M2) + .7123\ (B1) \\ & + .7545\ (B2) + .9511\ (S1) \\ & - .0000261\ (S1*S1) + 1615.6741 \end{aligned} \tag{23}$$

Since the various designs which are used in this study produce different coefficients for each design, the ratios of coefficients are different. Hence, the exact magnitude of the contribution of each factor as compared to other factors can not be determined with certainty. Nevertheless, the relative contribution of the factors can still be obtained and these contributions can be rank ordered.

Therefore, an accurate, but less definitive statement about the relative contribution of the two independent variables, is that the mean contribution of B2 is greater than the mean contribution of M1 to the response value. This conclusion is permissible because the relative magnitudes of the coefficients are maintained for the various designs even though the values of the coefficients

themselves change from one design to another. That is, no matter what orthogonal design is selected, the regression coefficient for B2 always exceeds the coefficient for M1. In this manner, the following conclusion is drawn for the five factor problem: the independent variable which causes the greatest change is S1, provided the values for S1 remain within the relevant range. This variable is followed in order of importance by: B2, B1, M2, and M1. Although this conclusion does not provide an exact measure of the difference between the factors, the relative ranking of the factors is still an important piece of information which the decision maker needs in order to evaluate his alternatives.

IV. CONCLUSION

OVERVIEW

This research identifies several experimental designs which are effective and efficient in estimating response surface equations for deterministic models. The specific deterministic model which is used in this study is a nuclear exchange linear programming model, but the results which are obtained in this study apply to any deterministic model.

The purpose of Response Surface Methodlogy in its application to any type of model is to obtain a response surface equation which closely approximates the true surface. A second order polynomial has been shown to be an excellent choice for estimating response surfaces of determininstic, nuclear exchange models. Ideally, the experimental design should use as few points as possible to obtain an accurate fit and the response equation should contain as few terms as possible. That is, the equation should contain as few cross product and squared terms as possible so that direct inferences can be drawn from the coefficients of the main effects.

SPECIFIC RESULTS

In keeping with the idea of simplifying the response equation, the response surface equations which are used to evaluate error in fit of the design are modified versions of the full regression equations. That is, selected terms from

the full regression equations have been omitted from the modified regression equations because these terms account for less than two percent of the total sum of squares.

In evaluating the response equations of these designs for their effectiveness, the results indicate that the accuracy of a modified response equation is dependent on the number of design points and on the degree of rotatability of the design. The accuracy of a response surface equation is highly correlated to the number of design points, but this relationship is not a perfect correlation, and their differences can be explained by the degree of rotatability of the designs.

In short, more design points improves accuracy, provided the points are spread throughout the entire surface. Similarly, if the region of interest is spherical, a rotatable design improves the accuracy of the response equation.

In evaluating these designs for their efficiency as well as effectiveness, the results indicate that a design which provides the most accurate fit with the fewest number of design points for all factors does not exist. In fact, minimizing the number of design points and maximizing accuracy are dichotomous goals. This trade-off between accuracy and required number of design points is dependent on the problem at hand and needs to be defined seperately for each situation. Therefore, it remains for the decision maker to evaluate the problem to determine which is more

important: a more accurate fit or fewer design points.

In evaluating the explanatory power of the response equations, the coefficients of the modified response equation represent marginal effects of change of the dependent variable when the independent variables are altered provided specific conditions are met. These conditions are: a high degree of multicollinearity does not exist between the independent variables; the main effect terms of the response equation dominate the squared and the interaction terms; and the modified response surface is able to accurately represent the true surface.

The results of this project indicate that there are limitations in the ability of the coefficients of the modified response equation to predict the marginal effects of change. The accuracy of these models to predict the marginal change can be evaluated by comparing the predicted changes of the coefficients of the modified response equations to the coefficients of the full regression equations. If this difference is unacceptable, then the full regression equation can be used to measure the marginal effects or the ratios of the coefficients of the modified equation can be used to determine the relative importance of the independent variables.

RECOMMENDATIONS

This study originally intended to examine the differences between bias minimizing and variance

minimizing designs. As this study evolved, it became clear that there were no significant differences between the two types of experimental designs, provided that the designs maintained certain characteristics, such as, rotatability and orthogonality.

However, very few designs possess pure rotatability or orthogonality. Most of the designs are classified in a gray area called, nearly rotatable and nearly orthogonal. As such, further research is required in specifying nearly rotatable and nearly orthogonal. This research may find what levels of rotatability are necessary to achieve a desired level of predictive power. Also, this research may find what levels of orthogonality are necessary to achieve a desired level of explanatory power.

Appendix A: MPOS Input Files

This appendix describes the specific problems which were analyzed for the three, four, five, and six factor cases. In addition, this appendix lists a sample of the Multi-Purpose Optimization System (MPOS) input files which were used to obtain the response value.

THREE FACTOR PROBLEM

There are three weapon types and five target classes. The three weapon types are:

WEAPON TYPE	LOW VALUE	HIGH VALUE
M1	1650	3000
B1	1200	3000
S1	6000	12000

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. The single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is provided in the following table:

TABLE A.1

SINGLE SHOT PROBABILITY OF KILL FOR THREE FACTOR PROBLEM

TARGET TYPE	WEAPON TYPE		
	M1	B1	S1
TGT CLASS 1	.84	.96	.73
TGT CLASS 2	.75	.88	.64
TGT CLASS 3	.55	.70	.32
TGT CLASS 4	.88	.98	.78
TGT CLASS 5	.25	.48	.15

Target class 4 represents time urgent targets and only ICBMs (M1) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 6000, 4000, 2000, 1000, and 200, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets.

FOUR FACTOR PROBLEM

There are four weapon types and five target classes. The four weapon types are:

WEAPON TYPE	LOW VALUE	HIGH VALUE
M1	1650	3000
M2	500	2000
B1	1200	3000
S1	6000	12000

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. The single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is provided in the following table:

TABLE A.2

SINGLE SHOT PROBABILITY OF KILL
FOR FOUR FACTOR PROBLEM

TARGET TYPE	WEAPON TYPE			
	M1	M2	B1	S1
TGT CLASS 1	.84	.90	.96	.73
TGT CLASS 2	.75	.83	.88	.64
TGT CLASS 3	.55	.62	.70	.32
TGT CLASS 4	.88	.94	.98	.78
TGT CLASS 5	.25	.36	.48	.15

Target class 4 represents time urgent targets and only ICBMs (M1, M2) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 6000, 4000, 2000, 1000, and 200, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets.

FIVE FACTOR PROBLEM

There are five weapon types and five target classes. The five weapon types are:

WEAPON TYPE	LOW VALUE	HIGH VALUE
M1	1650	3000
M2	500	2000
B1	1200	3000
B2	2000	4000
S1	6000	12000

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. The single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is provided in the following table:

TABLE A.3

SINGLE SHOT PROBABILITY OF KILL
FOR FIVE FACTOR PROBLEM

TARGET TYPE	WEAPON TYPE				
	M1	M2	B1	B2	S1
TGT CLASS 1	.84	.90	.96	.98	.73
TGT CLASS 2	.75	.83	.88	.92	.64
TGT CLASS 3	.55	.62	.70	.74	.32
TGT CLASS 4	.88	.94	.98	.99	.78
TGT CLASS 5	.25	.36	.48	.55	.15

Target class 4 represents time urgent targets and only ICBMs (M1, M2) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 9000, 6000, 3000, 1500, and 300, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets.

SIX FACTOR PROBLEM

There are six weapon types and five target classes. The six weapon types are:

WEAPON TYPE	LOW VALUE	HIGH VALUE
M1	1650	3000
M2	500	2000
M3	400	1000
B1	1200	3000
B2	2000	4000
S1	6000	12000

Assumption: Each system's reliability, probability of launch survivability, probability of arrival, and warhead reliability are all 1.0. the single shot probability of kill (which is damage expectancy on a target because of the previous assumptions of reliability, etc.) for each warhead type is:

TABLE A.4

SINGLE SHOT PROBABILITY OF KILL
FOR SIX FACTOR PROBLEM

TARGET TYPE	WEAPON TYPE					
	M1	M2	M3	B1	B2	S1
TGT CLASS 1	.84	.90	.92	.96	.98	.73
TGT CLASS 2	.75	.83	.87	.88	.92	.64
TGT CLASS 3	.55	.62	.65	.70	.74	.32
TGT CLASS 4	.88	.94	.95	.98	.99	.78
TGT CLASS 5	.25	.36	.45	.48	.55	.15

Target class 4 represents time urgent targets and only ICBMs (M1, M2, M3) and SLBMs (S1) are allowed to hit them. In addition, the maximum allowable damage on target class 2 targets is .95 of the total target value and at least .60 of the total target value of target class 5 targets must be destroyed. Furthermore, a maximum of two warheads can targeted at any individual target.

The number of targets in each class is 9000, 6000, 3000, 1500, and 300, respectively. The measure of effectiveness for this problem is the total damage expectancy, i.e., the sum of the product of the damage expectancy on a target times the number of targets.

```
*******************************************
************ MPOS INPUT FILE **************
************    3 FACTORS     **************
*******************************************

REGULAR
TITLE
RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

VARIABLES                            **  WEAPON **
                                     **  SYSTEM **
X11   TO X15                         ** M1      **
X21   TO X25                         ** B1      **
X31   TO X35                         ** S1      **
X41   TO X45                         ** M1 + M1 **
X51   TO X55                         ** B1 + B1 **
X61   TO X65                         ** S1 + S1 **
X71   TO X75                         ** M1 + B1 **
X81   TO X85                         ** M1 + S1 **
X91   TO X95                         ** B1 + S1 **


MAXIMIZE                             ** DE TABLE **

.8400 X11  + .7500 X12  + .5500 X13  + .8800 X14  + .2500 X15 +
.9600 X21  + .8800 X22  + .7000 X23  + .0000 X24  + .4800 X25 +
.7300 X31  + .6400 X32  + .3200 X33  + .7800 X34  + .1500 X35 +
.9744 X41  + .9375 X42  + .7975 X43  + .9856 X44  + .4375 X45 +
.9984 X51  + .9856 X52  + .9100 X53  + .0000 X54  + .7296 X55 +
.9271 X61  + .8704 X62  + .5376 X63  + .9516 X64  + .2775 X65 +
.9936 X71  + .9700 X72  + .8650 X73  + .0000 X74  + .6100 X75 +
.9568 X81  + .9100 X82  + .6940 X83  + .9736 X84  + .3625 X85 +
.9892 X91  + .9568 X92  + .7960 X93  + .0000 X94  + .5580 X95

CONSTRAINTS
** (1) 95 % DE CRITERIA **
.7500 X12  + .8800 X22  + .6400 X32  + .9375 X42  + .9856 X52  +
.8704 X62  + .9700 X72  + .9100 X82  + .9568 X92  .LE. 3800

** (2) 60 % DE CRITERIA **
.2500 X15  + .4800 X25  + .1500 X35  + .4375 X45  + .7296 X55  +
.2775 X65  + .6100 X75  + .3625 X85  + .5580 X95  .LE. 120

** (3) TARGET 1 CONSTRAINT **
X11  + X21  + X31  + X41  + X51  + X61  + X71 +
X81  + X91 .LE. 6000

** (4) TARGET 2 CONSTRAINT **
X12  + X22  + X32  + X42  + X52  + X62  + X72 +
X82  + X92 .LE. 4000
```

```
** (5) TARGET 3 CONSTRAINT **
X13   + X23   + X33   + X43   + X53   + X63   + X73 +
X83   + X93 .LE. 2000

** (6) TARGET 4 CONSTRAINT **
X14   + X24   + X34   + X44   + X54   + X64   + X74 +
X84   + X94 .LE. 1000

** (7) TARGET 5 CONSTRAINTS **
X15   + X25   + X35   + X45   + X55   + X65   + X75 +
X85   + X95 .LE. 200

** (8) WEAPON SYSTEM 1 (M1) CONSTRAINT **
X11   +  X12   + X13   + X14   + X15   +
2 X41   + 2 X42 + 2 X43 + 2 X44 + 2 X45 +
X71   + X72 + X73 + X74 + X75 +
X81   + X82 + X83 + X84 + X85 .LE. 3000

** (9) WEAPON SYSTEM 2 (B1) CONSTRAINT
X21   + X22   + X23   + X24   + X25   +
2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 +
X71   + X72   + X73   + X74   + X75   +
X91   + X92   + X93   + X94   + X95   .LE. 3000

** (10) WEAPON SYSTEM 3 (S1) CONSTRAINT **
X31   + X32   + X33   + X34   + X35   +
2 X61 + 2 X62 + 2 X63 + 2 X64 + 2 X65 +
X81   + X82   + X83   + X84   + X85   +
X91   + X92   + X93   + X94   + X95   .LE. 9000

PRINT
OPTIMIZE
```

```
******************************************
************ MPOS INPUT FILE *************
************     4 FACTOR    *************
******************************************

REGULAR
TITLE
RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

VARIABLES                                 **  WEAPON **
                                          **  SYSTEM **
X11   TO X15                              ** M1      **
X21   TO X25                              ** B1      **
X31   TO X35                              ** S1      **
X41   TO X45                              ** M1 + M1 **
X51   TO X55                              ** B1 + B1 **
X61   TO X65                              ** S1 + S1 **
X71   TO X75                              ** M1 + B1 **
X81   TO X85                              ** M1 + S1 **
X91   TO X95                              ** B1 + S1 **
X101  TO X105                             ** M2      **
X111  TO X115                             ** M2 + M2 **
X121  TO X125                             ** M1 + M2 **
X131  TO X135                             ** M2 + B1 **
X141  TO X145                             ** M2 + S1 **

MAXIMIZE                                  ** DE TABLE **

.8400 X11  + .7500 X12  + .5500 X13  + .8800 X14  + .2500 X15  +
.9600 X21  + .8800 X22  + .7000 X23  + .0000 X24  + .4800 X25  +
.7300 X31  + .6400 X32  + .3200 X33  + .7800 X34  + .1500 X35  +
.9744 X41  + .9375 X42  + .7975 X43  + .9856 X44  + .4375 X45  +
.9984 X51  + .9856 X52  + .9100 X53  + .0000 X54  + .7296 X55  +
.9271 X61  + .8704 X62  + .5376 X63  + .9516 X64  + .2775 X65  +
.9936 X71  + .9700 X72  + .8650 X73  + .0000 X74  + .6100 X75  +
.9568 X81  + .9100 X82  + .6940 X83  + .9736 X84  + .3625 X85  +
.9892 X91  + .9568 X92  + .7960 X93  + .0000 X94  + .5580 X95  +
.9000 X101 + .8300 X102 + .6200 X103 + .9400 X104 + .3600 X105 +
.9900 X111 + .9711 X112 + .8556 X113 + .9964 X114 + .5904 X115 +
.9840 X121 + .9575 X122 + .8290 X123 + .9928 X124 + .5200 X125 +
.9960 X131 + .9796 X132 + .8860 X133 + .0000 X134 + .6672 X135 +
.9730 X141 + .9388 X142 + .7416 X143 + .9868 X144 + .4560 X145

CONSTRAINTS
** (1) 95 % DE CRITERIA **
.7500 X12  + .8800 X22  + .6400 X32  + .9375 X42  + .9856 X52  +
.8704 X62  + .9700 X72  + .9100 X82  + .9568 X92  + .8300 X102 +
.9711 X112 + .9575 X122 + .9796 X132 + .9388 X142 .LE. 3800
```

```
** (2) 60 % DE CRITERIA **
.2500 X15  + .4800 X25  + .1500 X35  + .4375 X45  + .7296 X55  +
.2775 X65  + .6100 X75  + .3625 X85  + .5580 X95  + .3600 X105 +
.5904 X115 + .5200 X125 + .6672 X135 + .4560 X145 .GE. 120

** (3) TARGET 1 CONSTRAINT **
X11  + X21  + X31  + X41  + X51  + X61  + X71  + X81  +
X91  + X101 + X111 + X121 + X131 + X141 .LE. 6000

** (4) TARGET 2 CONSTRAINT **
X12  + X22  + X32  + X42  + X52  + X62  + X72  + X82  +
X92  + X102 + X112 + X122 + X132 + X142 .LE. 4000

** (5) TARGET 3 CONSTRAINT **
X13  + X23  + X33  + X43  + X53  + X63  + X73  + X83  +
X93  + X103 + X113 + X123 + X133 + X143 .LE. 2000

** (6) TARGET 4 CONSTRAINT **
X14  + X24  + X34  + X44  + X54  + X64  + X74  + X84  +

X94  + X104 + X114 + X124 + X134 + X144 .LE. 1000

** (7) TARGET 5 CONSTRAINTS **
X15  + X25  + X35  + X45  + X55  + X65  + X75  + X85  +

X95  + X105 + X115 + X125 + X135 + X145 .LE. 200

** (8) WEAPON SYSTEM 1 (M1) CONSTRAINT **
X11  +  X12  + X13  + X14  + X15  +
2 X41  + 2 X42 + 2 X43 + 2 X44 + 2 X45 +
X71  + X72 + X73 + X74 + X75 +
X81  + X82 + X83 + X84 + X85 +
X121 + X122 + X123 + X124 + X125 .LE. 3000

** (9) WEAPON SYSTEM 2 (M2) CONSTRAINT **
X101 + X102 + X103 + X104 + X105 +
2 X111 + 2 X112 + 2 X113 + 2 X114 + 2 X115 +
X121 + X122 + X123 + X124 + X125 +
X131 + X132 + X133 + X134 + X135 +
X141 + X142 + X143 + X144 + X145 .LE. 2000

** (10) WEAPON SYSTEM 3 (B1) CONSTRAINT
X21  + X22  + X23  + X24  + X25  +
2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 +
X71  + X72  + X73  + X74  + X75  +
X91  + X92  + X93  + X94  + X95  +
X131 + X132 + X133 + X134 + X135 .LE. 3000
```

```
** (11) WEAPON SYSTEM 4 (S1) CONSTRAINT **
X31   + X32   + X33   + X34   + X35   +
2 X61 + 2 X62 + 2 X63 + 2 X64 + 2 X65 +
X81   + X82   + X83   + X84   + X85   +
X91   + X92   + X93   + X94   + X95   +
X141 + X142 + X143 + X144 + X145 .LE. 9000

PRINT
OPTIMIZE
```

```
****************************************
************ MPOS INPUT FILE **************
************    5 FACTORS    **************
****************************************

REGULAR
TITLE
RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

VARIABLES                                   **  WEAPON **
                                            **  SYSTEM **
X11   TO X15                                ** M1      **
X21   TO X25                                ** B1      **
X31   TO X35                                ** S1      **
X41   TO X45                                ** M1 + M1 **
X51   TO X55                                ** B1 + B1 **
X61   TO X65                                ** S1 + S1 **
X71   TO X75                                ** M1 + B1 **
X81   TO X85                                ** M1 + S1 **
X91   TO X95                                ** B1 + S1 **
X101  TO X105                               ** M2      **
X111  TO X115                               ** M2 + M2 **
X121  TO X125                               ** M1 + M2 **
X131  TO X135                               ** M2 + B1 **
X141  TO X145                               ** M2 + S1 **
X151  TO X155                               ** B2      **
X161  TO X165                               ** B2 + B2 **
X171  TO X175                               ** M1 + B2 **
X181  TO X185                               ** M2 + B2 **
X191  TO X195                               ** B1 + B2 **
X201  TO X205                               ** B2 + S1 **

MAXIMIZE                                    ** DE TABLE **

.8400 X11  + .7500 X12  + .5500 X13  + .8800 X14  + .2500 X15 +
.9600 X21  + .8800 X22  + .7000 X23  + .0000 X24  + .4800 X25 +
.7300 X31  + .6400 X32  + .3200 X33  + .7800 X34  + .1500 X35 +
.9744 X41  + .9375 X42  + .7975 X43  + .9856 X44  + .4375 X45 +
.9984 X51  + .9856 X52  + .9100 X53  + .0000 X54  + .7296 X55 +
.9271 X61  + .8704 X62  + .5376 X63  + .9516 X64  + .2775 X65 +
.9936 X71  + .9700 X72  + .8650 X73  + .0000 X74  + .6100 X75 +
.9568 X81  + .9100 X82  + .6940 X83  + .9736 X84  + .3625 X85 +
.9892 X91  + .9568 X92  + .7960 X93  + .0000 X94  + .5580 X95 +
.9000 X101 + .8300 X102 + .6200 X103 + .9400 X104 + .3600 X105 +
.9900 X111 + .9711 X112 + .8556 X113 + .9964 X114 + .5904 X115 +
.9840 X121 + .9575 X122 + .8290 X123 + .9928 X124 + .5200 X125 +
.9960 X131 + .9796 X132 + .8860 X133 + .0000 X134 + .6672 X135 +
.9730 X141 + .9388 X142 + .7416 X143 + .9868 X144 + .4560 X145 +
.9800 X151 + .9200 X152 + .7400 X153 + .0000 X154 + .5500 X155 +
.9996 X161 + .9936 X162 + .9324 X163 + .0000 X164 + .7975 X165 +
.9968 X171 + .9936 X172 + .8830 X173 + .0000 X174 + .6625 X175 +
.9980 X181 + .9864 X182 + .9012 X183 + .0000 X184 + .7120 X185 +
.9992 X191 + .9904 X192 + .9220 X193 + .0000 X194 + .7660 X195 +
.9946 X201 + .9712 X202 + .8232 X203 + .0000 X204 + .6175 X205
```

```
CONSTRAINTS
** (1) 95 % DE CRITERIA **
.7500 X12  + .8800 X22  + .6400 X32  + .9375 X42  +
.9856 X52  + .8704 X62  + .9700 X72  + .9100 X82  +
.9568 X92  + .8300 X102 + .9711 X112 + .9575 X122 +
.9796 X132 + .9388 X142 + .9200 X152 + .9936 X162 +
.9936 X172 + .9864 X182 + .9904 X192 + .9712 X202 .LE. 5700

** (2) 60 % DE CRITERIA **
.2500 X15  + .4800 X25  + .1500 X35  + .4375 X45  +
.7296 X55  + .2775 X65  + .6100 X75  + .3625 X85  +
.5580 X95  + .3600 X105 + .5904 X115 + .5200 X125 +
.6672 X135 + .4560 X145 + .5500 X155 + .7975 X165 +
.6625 X175 + .7120 X185 + .7660 X195 + .6175 X205 .GE. 180

** (3) TARGET 1 CONSTRAINT **
X11  + X21  + X31  + X41  + X51  + X61  + X71  + X81  + X91  +
X101 + X111 + X121 + X131 + X141 + X151 + X161 + X171 + X181 +
X191 + X201 .LE. 9000

** (4) TARGET 2 CONSTRAINT **
X12  + X22  + X32  + X42  + X52  + X62  + X72  + X82  + X92  +
X102 + X112 + X122 + X132 + X142 + X152 + X162 + X172 + X182 +
X192 + X202 .LE. 6000

** (5) TARGET 3 CONSTRAINT **
X13  + X23  + X33  + X43  + X53  + X63  + X73  + X83  + X93  +
X103 + X113 + X123 + X133 + X143 + X153 + X163 + X173 + X183 +
X193 + X203 .LE. 3000

** (6) TARGET 4 CONSTRAINT **
X14  + X24  + X34  + X44  + X54  + X64  + X74  + X84  + X94  +
X104 + X114 + X124 + X134 + X144 + X154 + X164 + X174 + X184 +
X194 + X204 .LE. 1500

** (7) TARGET 5 CONSTRAINTS **
X15  + X25  + X35  + X45  + X55  + X65  + X75  + X85  + X95  +
X105 + X115 + X125 + X135 + X145 + X155 + X165 + X175 + X185 +
X195 + X205 .LE. 300

** (8) WEAPON SYSTEM 1 (M1) CONSTRAINT **
X11  +  X12  + X13  + X14  + X15  +
2 X41  + 2 X42 + 2 X43 + 2 X44 + 2 X45 +
X71  + X72 + X73 + X74 + X75 +
X81  + X82 + X83 + X84 + X85 +
X121 + X122 + X123 + X124 + X125 +
X171 + X172 + X173 + X174 + X175 .LE. 2325
```

```
** (9) WEAPON SYSTEM 2 (M2) CONSTRAINT **
X101 + X102 + X103 + X104 + X105 +
2 X111 + 2 X112 + 2 X113 + 2 X114 + 2 X115 +
X121 + X122 + X123 + X124 + X125 +
X131 + X132 + X133 + X134 + X135 +
X141 + X142 + X143 + X144 + X145 +
X181 + X182 + X183 + X184 + X185 . LE. 1250

** (10) WEAPON SYSTEM 3 (B1) CONSTRAINT
X21  + X22  + X23  + X24  + X25  +
2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 +
X71  + X72  + X73  + X74  + X75  +
X91  + X92  + X93  + X94  + X95  +
X131 + X132 + X133 + X134 + X135 +
X191 + X192 + X193 + X194 + X195 .LE. 2100

** (11) WEAPON SYSTEM 4 (B2) CONSTRAINT **
X151 + X152 + X153 + X154 + X155 +
2 X161 + 2 X162 + 2 X163 + 2 X164 + 2 X165 +
X171 + X172 + X173 + X174 + X175 +
X181 + X182 + X183 + X184 + X185 +
X191 + X192 + X193 + X194 + X195 +
X201 + X202 + X203 + X204 + X205 .LE. 3000

** (12) WEAPON SYSTEM 5 (S1) CONSTRAINT **
X31  + X32  + X33  + X34  + X35  +
2 X61 + 2 X62 + 2 X63 + 2 X64 + 2 X65 +
X81  + X82  + X83  + X84  + X85  +
X91  + X92  + X93  + X94  + X95  +
X141 + X142 + X143 + X144 + X145 +
X201 + X202 + X203 + X204 + X205 .LE. 9000

PRINT
OPTIMIZE
```

```
*******************************************
************ MPOS INPUT FILE **************
************    6 FACTORS     **************
*******************************************

REGULAR
TITLE
RESPONSE SURFACE FOR ARSENAL EXCHANGE MODEL

VARIABLES                              **  WEAPON **
                                       **  SYSTEM **
X11   TO X15                           ** M1      **
X21   TO X25                           ** B1      **
X31   TO X35                           ** S1      **
X41   TO X45                           ** M1 + M1 **
X51   TO X55                           ** B1 + B1 **
X61   TO X65                           ** S1 + S1 **
X71   TO X75                           ** M1 + B1 **
X81   TO X85                           ** M1 + S1 **
X91   TO X95                           ** B1 + S1 **
X101  TO X105                          ** M2      **
X111  TO X115                          ** M2 + M2 **
X121  TO X125                          ** M1 + M2 **
X131  TO X135                          ** M2 + B1 **
X141  TO X145                          ** M2 + S1 **
X151  TO X155                          ** B2      **
X161  TO X165                          ** B2 + B2 **
X171  TO X175                          ** M1 + B2 **
X181  TO X185                          ** M2 + B2 **
X191  TO X195                          ** B1 + B2 **
X201  TO X205                          ** B2 + S1 **
X211  TO X215                          ** M3      **
X221  TO X225                          ** M3 + M3 **
X231  TO X235                          ** M1 + M3 **
X241  TO X245                          ** M2 + M3 **
X251  TO X255                          ** B1 + M3 **
X261  TO X265                          ** B2 + M3 **
X271  TO X275                          ** M3 + S1 **
```

MAXIMIZE ** DE TABLE **

```
.8400 X11  + .7500 X12  + .5500 X13  + .8800 X14  + .2500 X15  +
.9600 X21  + .8800 X22  + .7000 X23  + .0000 X24  + .4800 X25  +
.7300 X31  + .6400 X32  + .3200 X33  + .7800 X34  + .1500 X35  +
.9744 X41  + .9375 X42  + .7975 X43  + .9856 X44  + .4375 X45  +
.9984 X51  + .9856 X52  + .9100 X53  + .0000 X54  + .7296 X55  +
.9271 X61  + .8704 X62  + .5376 X63  + .9516 X64  + .2775 X65  +
.9936 X71  + .9700 X72  + .8650 X73  + .0000 X74  + .6100 X75  +
.9568 X81  + .9100 X82  + .6940 X83  + .9736 X84  + .3625 X85  +
.9892 X91  + .9568 X92  + .7960 X93  + .0000 X94  + .5580 X95  +
.9000 X101 + .8300 X102 + .6200 X103 + .9400 X104 + .3600 X105 +
.9900 X111 + .9711 X112 + .8556 X113 + .9964 X114 + .5904 X115 +
.9840 X121 + .9575 X122 + .8290 X123 + .9928 X124 + .5200 X125 +
.9960 X131 + .9796 X132 + .8860 X133 + .0000 X134 + .6672 X135 +
.9730 X141 + .9388 X142 + .7416 X143 + .9868 X144 + .4560 X145 +
.9800 X151 + .9200 X152 + .7400 X153 + .0000 X154 + .5500 X155 +
.9996 X161 + .9936 X162 + .9324 X163 + .0000 X164 + .7975 X165 +
.9968 X171 + .9936 X172 + .8830 X173 + .0000 X174 + .6625 X175 +
.9980 X181 + .9864 X182 + .9012 X183 + .0000 X184 + .7120 X185 +
.9992 X191 + .9904 X192 + .9220 X193 + .0000 X194 + .7660 X195 +
.9946 X201 + .9712 X202 + .8232 X203 + .0000 X204 + .6175 X205 +
.9200 X211 + .8700 X212 + .6500 X213 + .9500 X214 + .4500 X215 +
.9936 X221 + .9831 X222 + .8775 X223 + .9975 X224 + .6975 X225 +
.9872 X231 + .9675 X232 + .8425 X233 + .9940 X234 + .5875 X235 +
.9920 X241 + .9779 X242 + .8670 X243 + .9970 X244 + .6480 X245 +
.9968 X251 + .9844 X252 + .8950 X253 + .0000 X254 + .7140 X255 +
.9984 X261 + .9896 X262 + .9090 X263 + .0000 X264 + .7525 X265 +
.9784 X271 + .9532 X272 + .7620 X273 + .9890 X274 + .5325 X275
```

CONSTRAINTS

```
** (1) 95 % DE CRITERIA **
.7500 X12  + .8800 X22  + .6400 X32  + .9375 X42  + .9856 X52  +
.8704 X62  + .9700 X72  + .9100 X82  + .9568 X92  + .8300 X102 +
.9711 X112 + .9575 X122 + .9796 X132 + .9388 X142 + .9200 X152 +
.9936 X162 + .9936 X172 + .9864 X182 + .9904 X192 + .9712 X202 +
.8700 X212 + .9831 X222 + .9675 X232 + .9779 X242 + .9844 X252 +
.9896 X262 + .9532 X272 .LE. 5700

** (2) 60 % DE CRITERIA **
.2500 X15  + .4800 X25  + .1500 X35  + .4375 X45  + .7296 X55  +
.2775 X65  + .6100 X75  + .3625 X85  + .5580 X95  + .3600 X105 +
.5904 X115 + .5200 X125 + .6672 X135 + .4560 X145 + .5500 X155 +
.7975 X165 + .6625 X175 + .7120 X185 + .7660 X195 + .6175 X205 +
.4500 X215 + .6975 X225 + .5875 X235 + .6480 X245 + .7140 X255 +
.7525 X265 + .5325 X275 .GE. 180
```

```
** (3) TARGET 1 CONSTRAINT **
X11  + X21  + X31  + X41  + X51  + X61  + X71  + X81  +
X91  + X101 + X111 + X121 + X131 + X141 + X151 + X161 +
X171 + X181 + X191 + X201 + X211 + X221 + X231 + X241 +
X251 + X261 + X271 .LE. 9000

** (4) TARGET 2 CONSTRAINT **
X12  + X22  + X32  + X42  + X52  + X62  + X72  + X82  +
X92  + X102 + X112 + X122 + X132 + X142 + X152 + X162 +
X172 + X182 + X192 + X202 + X212 + X222 + X232 + X242 +
X252 + X262 + X272 .LE. 6000

** (5) TARGET 3 CONSTRAINT **
X13  + X23  + X33  + X43  + X53  + X63  + X73  + X83  +
X93  + X103 + X113 + X123 + X133 + X143 + X153 + X163 +
X173 + X183 + X193 + X203 + X213 + X223 + X233 + X243 +
X253 + X263 + X273 .LE. 3000

** (6) TARGET 4 CONSTRAINT **
X14  + X24  + X34  + X44  + X54  + X64  + X74  + X84  +
X94  + X104 + X114 + X124 + X134 + X144 + X154 + X164 +
X174 + X184 + X194 + X204 + X214 + X224 + X234 + X244 +
X254 + X264 + X274 .LE. 1500

** (7) TARGET 5 CONSTRAINTS **
X15  + X25  + X35  + X45  + X55  + X65  + X75  + X85  +
X95  + X105 + X115 + X125 + X135 + X145 + X155 + X165 +
X175 + X185 + X195 + X205 + X215 + X225 + X235 + X245 +
X255 + X265 + X275 .LE. 300

** (8) WEAPON SYSTEM 1 (M1) CONSTRAINT **
X11   +  X12  + X13  + X14  + X15  +
2 X41  + 2 X42 + 2 X43 + 2 X44 + 2 X45 +
X71  + X72 + X73 + X74 + X75 +
X81  + X82 + X83 + X84 + X85 +
X121 + X122 + X123 + X124 + X125 +
X171 + X172 + X173 + X174 + X175 +
X231 + X232 + X233 + X234 + X235 .LE. 2325

** (9) WEAPON SYSTEM 2 (M2) CONSTRAINT **
X101 + X102 + X103 + X104 + X105 +
2 X111 + 2 X112 + 2 X113 + 2 X114 + 2 X115 +
X121 + X122 + X123 + X124 + X125 +
X131 + X132 + X133 + X134 + X135 +
X141 + X142 + X143 + X144 + X145 +
X181 + X182 + X183 + X184 + X185 +
X241 + X242 + X243 + X244 + X245 .LE. 1250
```

```
** (10) WEAPON SYSTEM 3 (M3) CONSTRAINT **
X211 + X212 + X213 + X214 + X215 +
2 X221 + 2 X222 + 2 X223 + 2 X224 + 2 X225 +
X231 + X232 + X233 + X234 + X235 +
X241 + X242 + X243 + X244 + X245 +
X251 + X252 + X253 + X254 + X255 +
X261 + X262 + X263 + X264 + X265 +
X271 + X272 + X273 + X274 + X275 .LE. 700

** (11) WEAPON SYSTEM 4 (B1) CONSTRAINT
X21  + X22  + X23  + X24  + X25  +
2 X51 + 2 X52 + 2 X53 + 2 X54 + 2 X55 +
X71  + X72  + X73  + X74  + X75  +
X91  + X92  + X93  + X94  + X95  +
X131 + X132 + X133 + X134 + X135 +
X191 + X192 + X193 + X194 + X195 +
X251 + X252 + X253 + X254 + X255 .LE. 2100

** (12) WEAPON SYSTEM 5 (B2) CONSTRAINT **
X151 + X152 + X153 + X154 + X155 +
2 X161 + 2 X162 + 2 X163 + 2 X164 + 2 X165 +
X171 + X172 + X173 + X174 + X175 +
X181 + X182 + X183 + X184 + X185 +
X191 + X192 + X193 + X194 + X195 +
X201 + X202 + X203 + X204 + X205 +
X261 + X262 + X263 + X264 + X265 .LE. 3000

** (13) WEAPON SYSTEM 6 (S1) CONSTRAINT **
X31  + X32  + X33  + X34  + X35  +
2 X61 + 2 X62 + 2 X63 + 2 X64 + 2 X65 +
X81  + X82  + X83  + X84  + X85  +
X91  + X92  + X93  + X94  + X95  +
X141 + X142 + X143 + X144 + X145 +
X201 + X202 + X203 + X204 + X205 +
X271 + X272 + X273 + X274 + X275 .LE. 9000

PRINT
OPTIMIZE
```

Appendix B: Experimental Designs

TABLE B.1

BOX-BEHNKEN EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	3000	3000	9000	1	1	0	10131.4892
2	3000	1200	9000	1	-1	0	9242.6923
3	1650	3000	9000	-1	1	0	9671.9492
4	1650	1200	9000	-1	-1	0	8493.1923
5	3000	2100	12000	1	0	1	10395.4062
6	3000	2100	6000	1	0	-1	8477.3846
7	1650	2100	12000	-1	0	1	9935.8662
8	1650	2100	6000	-1	0	-1	7464.8846
9	2325	3000	12000	0	1	1	10584.9202
10	2325	3000	6000	0	1	-1	8757.9423
11	2325	1200	12000	0	-1	1	9724.2762
12	2325	1200	6000	0	-1	-1	7179.1346
13	2325	2100	9000	0	0	0	9474.4362

TABLE B.2

BOX-DRAPER EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	2914	2886	11619	0.873	0.873	0.873	10642.0176
2	2914	2886	6381	0.873	0.873	-0.873	9189.1323
3	2914	1314	11619	0.873	-0.873	0.873	9892.8949
4	2914	1314	6381	0.873	-0.873	-0.873	7965.0446
5	1736	2886	11619	-0.873	0.873	0.873	10251.0156
6	1736	2886	6381	-0.873	0.873	-0.873	8464.9046
7	1736	1314	11619	-0.873	-0.873	0.873	9491.9037
8	1736	1314	6381	-0.873	-0.873	-0.873	7081.5446
9	1334	2100	9000	-1.468	0	0	8943.0723
10	3316	2100	9000	1.468	0	0	9811.7725
11	2325	779	9000	0	-1.468	0	8576.7423
12	2325	3421	9000	0	1.468	0	10099.7576
13	2325	2100	4596	0	0	-1.468	7071.7846
14	2325	2100	13404	0	0	1.468	10471.3739
15	2325	2100	9000	0	0	0	9474.4362

TABLE B.3

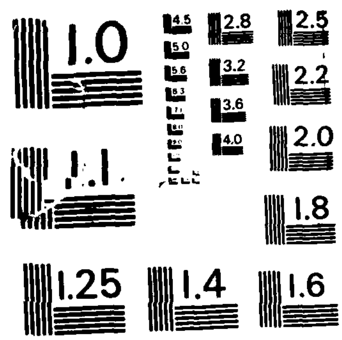
MINIMUM POINT EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	1650	1200	6000	-1	-1	-1	6672.8846
2	3000	1200	6000	1	-1	-1	7685.3846
3	1650	3000	6000	-1	1	-1	8256.8846
4	1650	1200	12000	-1	-1	1	9494.5061
5	2457	2280	6000	0.1925	0.1925	-1	8228.5346
6	2457	1200	9600	0.1925	-1	0.1925	9208.0423
7	1650	2280	9600	-1	0.1925	0.1925	9471.1782
8	2127	3000	12000	-0.2912	1	1	10520.1731
9	3000	1836	12000	1	-0.2912	1	10265.9406
10	3000	3000	8100	1	1	-0.2912	9924.1292

AD-A167 604 EVALUATING EXPERIMENTAL DESIGNS FOR FITTING RESPONSE SURFACES OF DETERMINISTIC MODELS(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI

2/2

UNCLASSIFIED B K ISHIHARA DEC 85 AFIT/GOR/OS/85D-10 F/G 12/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

TABLE B.4

KOSHAL EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	1988	1650	7500	-0.5	-0.5	-0.5	8282.3846
2	2662	1650	7500	0.5	-0.5	-0.5	8695.0323
3	1988	2550	7500	-0.5	0.5	-0.5	8940.8523
4	1988	1650	10500	-0.5	-0.5	0.5	9484.6413
5	3337	1650	7500	1.5	-0.5	-0.5	9079.7823
6	1988	3450	7500	-0.5	1.5	-0.5	9570.8523
7	1988	1650	13500	-0.5	-0.5	1.5	10175.8413
8	2662	2550	7500	0.5	0.5	-0.5	9325.0323
9	2662	1650	10500	0.5	-0.5	0.5	9714.0709
10	1988	2550	10500	-0.5	0.5	0.5	9920.9244

TABLE B.5

CENTRAL COMPOSITE EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	3000	3000	12000	1	1	1	10792.2127
2	3000	3000	6000	1	1	-1	9142.6923
3	3000	1200	12000	1	-1	1	9954.0461
4	3000	1200	6000	1	-1	-1	7685.3846
5	1650	3000	12000	-1	1	1	10362.5723
6	1650	3000	6000	-1	1	-1	8256.8846
7	1650	1200	12000	-1	-1	1	9494.5061
8	1650	1200	6000	-1	-1	-1	6672.8846
9	3460	2100	9000	1.682	0	0	9860.7901
10	1190	2100	9000	-1.682	0	0	8860.9923
11	2325	3614	9000	0	1.682	0	9978.8648
12	2325	586	9000	0	-1.682	0	8441.6423
13	2325	2100	14046	0	0	1.682	10597.9122
14	2325	2100	3954	0	0	-1.682	6654.4846
15	2325	2100	9000	0	0	0	9474.4362

TABLE B.6

HYBRID (311A) EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	2325	2100	15000	0	0	2	10785.9456
2	2325	2100	3000	0	0	-2	6034.3846
3	1370	827	12000	-1.414	-1.414	1	9204.3765
4	3280	827	12000	1.414	-1.414	1	9866.4389
5	1370	3373	12000	-1.414	1.414	1	10438.0595
6	3280	3373	12000	1.414	1.414	1	11037.5089
7	3675	2100	6000	2	0	-1	8897.4423
8	975	2100	6000	-2	0	-1	6958.3846
9	2325	3900	6000	0	2	-1	9387.9423
10	2325	300	6000	0	-2	-1	6387.1346
11	2325	2100	9000	0	0	0	9474.4362

TABLE B.7

HYBRID (310+CP) EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	2325	2100	12872	0	0	1.2906	10366.5168
2	2325	2100	8592	0	0	-0.136	9320.2623
3	1650	1200	10916	-1	-1	0.6386	9244.7525
4	3000	1200	10916	1	-1	0.6386	9704.2925
5	1650	3000	10916	-1	1	0.6386	10113.3956
6	3000	3000	10916	1	1	0.6386	10569.6987
7	3117	2100	6218	1.1736	0	-0.9273	8679.6623
8	1533	2100	6218	-1.1736	0	-0.9273	7516.6546
9	2325	3156	6218	0	1.1736	-0.9273	8967.4223
10	2325	1044	6218	0	-1.1736	-0.9273	7181.3746
11	2325	2100	9000	0	0	0	9474.4361

TABLE B.8

FULL FACTORIAL EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
1	3000	3000	12000	1	1	1	10792.2129
2	3000	3000	9000	1	1	0	10131.4893
3	3000	3000	6000	1	1	-1	9142.6924
4	3000	2100	12000	1	0	1	10395.4063
5	3000	2100	9000	1	0	0	9704.2061
6	3000	2100	6000	1	0	-1	8477.3848
7	3000	1200	12000	1	-1	1	9954.0459
8	3000	1200	9000	1	-1	0	9242.6924
9	3000	1200	6000	1	-1	-1	7685.3848
10	2325	3000	12000	0	1	1	10534.9199
11	2325	3000	9000	0	1	0	9901.7188
12	2325	3000	6000	0	1	-1	8757.9424
13	2325	2100	12000	0	0	1	10165.6357
14	2325	2100	9000	0	0	0	9474.4365
15	2325	2100	6000	0	0	-1	7971.1348
16	2325	1200	12000	0	-1	1	9724.2764
17	2325	1200	9000	0	-1	0	8871.4424
18	2325	1200	6000	0	-1	-1	7179.1348

TABLE B.8 (continued)

FULL FACTORIAL EXPERIMENTAL DESIGN FOR THREE FACTORS

	NON-CODED			CODED			
RUN#	M1	B1	S1	M1	B1	S1	RESPONSE
19	1650	3000	12000	-1	1	1	10362.5723
20	1650	3000	9000	-1	1	0	9671.9492
21	1650	3000	6000	-1	1	-1	8256.8848
22	1650	2100	12000	-1	0	1	9935.8662
23	1650	2100	9000	-1	0	0	9123.1924
24	1650	2100	6000	-1	0	-1	7464.8848
25	1650	1200	12000	-1	-1	1	9494.5059
26	1650	1200	9000	-1	-1	0	8493.1924
27	1650	1200	6000	-1	-1	-1	6672.8848

TABLE B.9

BOX-BEHNKEN EXPERIMENTAL DESIGN FOR FOUR FACTORS

	NON-CODED				CODED				
RUN#	M1	M2	B1	S1	M1	M2	B1	S1	RESPONSE
1	3000	2000	2100	9000	1	1	0	0	10530.8908
2	3000	500	2100	9000	1	-1	0	0	9914.4062
3	1650	2000	2100	9000	-1	1	0	0	10085.4662
4	1650	500	2100	9000	-1	-1	0	0	9445.1923
5	2325	1250	3000	12000	0	0	1	1	11043.7952
6	2325	1250	3000	6000	0	0	1	-1	9570.4423
7	2325	1250	1200	12000	0	0	-1	1	10242.4877
8	2325	1250	1200	6000	0	0	-1	-1	8216.6346
9	3000	1250	2100	12000	1	0	0	1	10862.7746
10	3000	1250	2100	6000	1	0	0	-1	9325.1923
11	1650	1250	2100	12000	-1	0	0	1	10444.2508
12	1650	1250	2100	6000	-1	0	0	-1	8502.3846
13	2325	2000	3000	9000	0	1	1	0	10714.9038
14	2325	2000	1200	9000	0	1	-1	0	9873.8762
15	2325	500	3000	9000	0	-1	1	0	10111.9192
16	2325	500	1200	9000	0	-1	-1	0	9186.4423
17	3000	1250	3000	9000	1	0	1	0	10645.8738
18	3000	1250	1200	9000	1	0	-1	0	9788.3462
19	1650	1250	3000	9000	-1	0	1	0	10197.4492
20	1650	1250	1200	9000	-1	0	-1	0	9287.6923
21	2325	2000	2100	12000	0	1	0	1	10930.8071
22	2325	2000	2100	6000	0	1	0	-1	9427.9423
23	2325	500	2100	12000	0	-1	0	1	10374.5477
24	2325	500	2100	6000	0	-1	0	-1	8386.1346
25	2325	1250	2100	9000	0	0	0	0	9999.9361

TABLE B.10

BOX-DRAPER EXPERIMENTAL DESIGN FOR FOUR FACTORS

	NON-CODED				CODED				
RUN#	M1	M2	B1	S1	M1	M2	B1	S1	RESPONSE
1	2781	1756	2707	11025	.675	.675	.675	.675	11052.2726
2	2781	1756	2707	6975	.675	.675	.675	-.675	10190.7768
3	2781	1756	1493	11025	.675	.675	-.675	.675	10514.5027
4	2781	1756	1493	6975	.675	.675	-.675	-.675	9542.6223
5	2781	744	2707	11025	.675	-.675	.675	.675	10680.2466
6	2781	744	2707	6975	.675	-.675	.675	-.675	9744.8623
7	2781	744	1493	11025	.675	-.675	-.675	.675	10111.3233
8	2781	744	1493	6975	.675	-.675	-.675	-.675	8895.0623
9	1869	1756	2707	11025	-.675	.675	.675	.675	10772.1974
10	1869	1756	2707	6975	-.675	.675	.675	-.675	9880.3320
11	1869	1756	1493	11025	-.675	.675	-.675	.675	10220.2949
12	1869	1756	1493	6975	-.675	.675	-.675	-.675	9033.0223
13	1869	744	2707	11025	-.675	-.675	.675	.675	10385.9703
14	1869	744	2707	6975	-.675	-.675	.675	-.675	9225.0223
15	1869	744	1493	11025	-.675	-.675	-.675	.675	9800.8786
16	1869	744	1493	6975	-.675	-.675	-.675	-.675	8336.4946
17	1414	1250	2100	9000	-1.349	0	0	0	9689.8318
18	3236	1250	2100	9000	1.349	0	0	0	10310.0405
19	2325	238	2100	9000	0	-1.349	0	0	9574.4913
20	2325	2262	2100	9000	0	1.349	0	0	10414.2856
21	2325	1250	886	9000	0	0	-1.349	0	9404.5905
22	2325	1250	3314	9000	0	0	1.349	0	10566.3179
23	2325	1250	2100	4953	0	0	0	-1.349	8338.5546
24	2325	1250	2100	13047	0	0	0	1.349	10861.8458
25	2325	1250	2100	9000	0	0	0	0	9999.9361

TABLE B.11

MINIMUM POINT EXPERIMENTAL DESIGN FOR FOUR FACTORS

	NON-CODED				CODED				
RUN#	M1	M2	B1	S1	M1	M2	B1	S1	RESPONSE
1	1650	500	1200	6000	-1	-1	-1	-1	7087.8846
2	3000	500	1200	6000	1	-1	-1	-1	8100.3846
3	1650	2000	1200	6000	-1	1	-1	-1	8332.8846
4	1650	500	3000	6000	-1	-1	1	-1	8671.8846
5	1650	500	1200	12000	-1	-1	-1	1	9704.7061
6	2604	1560	1200	6000	.4114	.4114	-1	-1	8690.9723
7	2604	500	2472	6000	.4114	-1	.4114	-1	8872.3723
8	2604	500	1200	10500	.4114	-1	-1	.4114	9683.8477
9	1650	1560	2472	6000	-1	.4114	.4114	-1	9017.5923
10	1650	1560	1200	10500	-1	.4114	-1	.4114	9804.7301
11	1650	500	2472	10500	-1	-1	.4114	.4114	9979.3780
12	1886	2000	3000	12000	-.6502	1	1	1	11181.9322
13	3000	760	3000	12000	1	-.6502	1	1	11071.2087
14	3000	2000	1512	12000	1	1	-.6502	1	10875.2048
15	3000	2000	3000	6900	1	1	1	-.6502	10485.3723

TABLE B.12

KOSHAL EXPERIMENTAL DESIGN FOR FOUR FACTORS

	NON-CODED				CODED				
RUN#	M1	M2	B1	S1	M1	M2	B1	S1	RESPONSE
1	2055	950	1740	7800	-.4	-.4	-.4	-.4	9167.5423
2	2730	950	1740	7800	.6	-.4	-.4	-.4	9542.6923
3	2055	1700	1740	7800	-.4	.6	-.4	-.4	9643.9423
4	2055	950	2640	7800	-.4	-.4	.6	-.4	9763.3672
5	2055	950	1740	10800	-.4	-.4	-.4	.6	10020.0841
6	3405	950	1740	7800	1.6	-.4	-.4	-.4	9788.4241
7	2055	2450	1740	7800	-.4	1.6	-.4	-.4	9959.4841
8	2055	950	3540	7800	-.4	-.4	1.6	-.4	10186.7272
9	2055	950	1740	13800	-.4	-.4	-.4	1.6	10656.2591
10	2730	1700	1740	7800	.6	.6	-.4	-.4	9873.9541
11	2730	950	2640	7800	.6	-.4	.6	-.4	9993.1372
12	2730	950	1740	10800	.6	-.4	-.4	.6	10249.8541
13	2055	1700	2640	7800	-.4	.6	.6	-.4	10078.6672
14	2055	1700	1740	10800	-.4	.6	-.4	.6	10326.9688
15	2055	950	2640	10800	-.4	-.4	.6	.6	10449.1518

TABLE B.13

CENTRAL COMPOSITE EXPERIMENTAL DESIGN FOR FOUR FACTORS

	NON-CODED				CODED				
RUN#	M1	M2	B1	S1	M1	M2	B1	S1	RESPONSE
1	3000	2000	3000	12000	1	1	1	1	11453.3481
2	3000	2000	3000	6000	1	1	1	-1	10281.0892
3	3000	2000	1200	12000	1	1	-1	1	10735.7096
4	3000	2000	1200	6000	1	1	-1	-1	9182.6923
5	3000	500	3000	12000	1	-1	1	1	10975.7627
6	3000	500	3000	6000	1	-1	1	-1	9467.6923
7	3000	500	1200	12000	1	-1	-1	1	10164.2461
8	3000	500	1200	6000	1	-1	-1	-1	8100.3846
9	1650	2000	3000	12000	-1	1	1	1	11111.8277
10	1650	2000	3000	6000	-1	1	1	-1	9673.1923
11	1650	2000	1200	12000	-1	1	-1	1	10315.1908
12	1650	2000	1200	6000	-1	1	-1	-1	8332.8846
13	1650	500	3000	12000	-1	-1	1	1	10559.2338
14	1650	500	3000	6000	-1	-1	1	-1	8671.8846
15	1650	500	1200	12000	-1	-1	-1	1	9704.7061
16	1650	500	1200	6000	-1	-1	-1	-1	7087.8846
17	975	1250	2100	9000	-2	0	0	0	9540.3961
18	3675	1250	2100	9000	2	0	0	0	10459.4762
19	2325	0	2100	9000	0	-2	0	0	9474.4362
20	2325	2750	2100	9000	0	2	0	0	10599.9208
21	2325	1250	300	9000	0	0	-2	0	9028.9423
22	2325	1250	3900	9000	0	0	2	0	10825.9638
23	2325	1250	2100	3000	0	0	0	-2	7071.8846
24	2325	1250	2100	15000	0	0	0	2	11239.2052
25	2325	1250	2100	9000	0	0	0	0	9999.9361

TABLE B.14

HYBRID (416A+CP) EXPERIMENTAL DESIGN FOR FOUR FACTORS

	NON-CODED				CODED				
RUN#	M1	M2	B1	S1	M1	M2	B1	S1	RESPONSE
1	2325	1250	2100	14353	0	0	0	1.7844	11118.1515
2	2325	1250	2100	4517	0	0	0	-1.4945	8057.9946
3	1650	500	1200	10932	-1	-1	-1	.644	9458.6389
4	3000	500	1200	10932	1	-1	-1	.644	9918.1789
5	1650	2000	1200	10932	-1	1	-1	.644	10084.3805
6	3000	2000	1200	10932	1	1	-1	.644	10523.0236
7	1650	500	3000	10932	-1	-1	1	.644	10327.2820
8	3000	500	3000	10932	1	-1	1	.644	10765.2599
9	1650	2000	3000	10932	-1	[illegible]	1	.644	10901.3249
10	3000	2000	3000	10932	1	1	1	.644	11290.6917
11	3463	1250	2100	6278	1.6853	0	0	-.9075	9716.9823
12	1187	1250	2100	6278	-1.6853	0	0	-.9075	8333.0546
13	2325	2514	2100	6278	0	1.6853	0	-.9075	9887.5823
14	2325	0	2100	6278	0	-1.6853	0	-.9075	8149.0546
15	2325	1250	3617	6278	0	0	1.6853	-.9075	10090.3072
16	2325	1250	583	6278	0	0	-1.6853	-.9075	7851.5946
17	2325	1250	2100	9000	0	0	0	0	9999.9361

TABLE B.15

HYBRID (416C) EXPERIMENTAL DESIGN FOR FOUR FACTORS

	NON-CODED				CODED				
RUN#	M1	M2	B1	S1	M1	M2	B1	S1	RESPONSE
1	2325	1250	2100	14296	0	0	0	1.7654	11107.4868
2	2325	1250	2100	9000	0	0	0	0	9999.9361
3	1650	500	1200	10702	-1	-1	-1	.5675	9405.6469
4	3000	500	1200	10702	1	-1	-1	.5675	9865.1869
5	1650	2000	1200	10702	-1	1	-1	.5675	10033.6885
6	3000	2000	1200	10702	1	1	-1	.5675	10474.6316
7	1650	500	3000	10702	-1	-1	1	.5675	10274.2900
8	3000	500	3000	10702	1	-1	1	.5675	10718.6746
9	1650	2000	3000	10702	-1	1	1	.5675	10855.9919
10	3000	2000	3000	10702	1	1	1	.5675	11255.6627
11	3317	1250	2100	5847	1.4697	0	0	-1.0509	9435.5023
12	1383	1250	2100	5847	-1.4697	0	0	-1.0509	8166.7146
13	2325	2352	2100	5847	0	1.4697	0	-1.0509	9586.3623
14	2325	148	2100	5847	0	-1.4697	0	-1.0509	7996.0546
15	2325	1250	3423	5847	0	0	1.4697	-1.0509	9796.1623
16	2325	1250	777	5847	0	0	-1.4697	-1.0509	7746.4746

TABLE B.16

BOX-BEHNKEN EXPERIMENTAL DESIGN FOR FIVE FACTORS

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
1	3000	2000	2100	3000	9000	1	1	0	0	0	14743.1514
2	3000	500	2100	3000	9000	1	-1	0	0	0	13760.1514
3	1650	2000	2100	3000	9000	-1	1	0	0	0	13973.6514
4	1650	500	2100	3000	9000	-1	-1	0	0	0	12888.7422
5	2325	1250	3000	4000	9000	0	0	1	1	0	15240.9014
6	2325	1250	3000	2000	9000	0	0	1	-1	0	13760.9014
7	2325	1250	1200	4000	9000	0	0	-1	1	0	13980.9014
8	2325	1250	1200	2000	9000	0	0	-1	-1	0	12305.4922
9	2325	2000	2100	3000	12000	0	1	0	0	1	15379.2168
10	2325	2000	2100	3000	6000	0	1	0	0	-1	12697.4922
11	2325	500	2100	3000	12000	0	-1	0	0	1	14748.6172
12	2325	500	2100	3000	6000	0	-1	0	0	-1	11452.4922
13	3000	1250	3000	3000	9000	1	0	1	0	0	14885.6514
14	3000	1250	1200	3000	9000	1	0	-1	0	0	13625.6514
15	1650	1250	3000	3000	9000	-1	0	1	0	0	14116.1514
16	1650	1250	1200	3000	9000	-1	0	-1	0	0	12719.2422
17	2325	1250	2100	4000	12000	0	0	0	1	1	15574.3164
18	2325	1250	2100	4000	6000	0	0	0	1	-1	12994.9922

TABLE B.16 (continued)

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
19	2325	1250	2100	2000	12000	0	0	0	-1	1	14510.9014
20	2325	1250	2100	2000	6000	0	0	0	-1	-1	11154.9922
21	2325	2000	3000	3000	9000	0	1	1	0	0	14988.4014
22	2325	2000	1200	3000	9000	0	1	-1	0	0	13728.4014
23	2325	500	3000	3000	9000	0	-1	1	0	0	14813.4014
24	2325	500	1200	3000	9000	0	-1	-1	0	0	12602.9922
25	3000	1250	2100	4000	9000	1	0	0	1	0	14995.6514
26	3000	1250	2100	2000	9000	1	0	0	-1	0	13515.6514
27	1650	1250	2100	4000	9000	-1	0	0	1	0	14226.1514
28	1650	1250	2100	2000	9000	-1	0	0	-1	0	12591.2422
29	2325	1250	3000	3000	12000	0	0	1	0	1	15487.2764
30	2325	1250	3000	3000	6000	0	0	1	0	-1	12867.7422
31	2325	1250	1200	3000	12000	0	0	-1	0	1	14620.9014
32	2325	1250	1200	3000	6000	0	0	-1	0	-1	11273.9922
33	3000	1250	2100	3000	12000	1	0	0	0	1	15293.6865
34	3000	1250	2100	3000	6000	1	0	0	0	-1	12588.7422
35	1650	1250	2100	3000	12000	-1	0	0	0	1	14834.1465
36	1650	1250	2100	3000	6000	-1	0	0	0	-1	11555.2422
37	2325	2000	2100	4000	9000	0	1	0	1	0	15098.4014
38	2325	2000	2100	2000	9000	0	1	0	-1	0	13618.4014
39	2325	500	2100	4000	9000	0	-1	0	1	0	14123.4014
40	2325	500	2100	2000	9000	0	-1	0	-1	0	12474.9922
41	2325	1250	2100	3000	9000	0	0	0	0	0	13870.9014

TABLE B.17

BOX-DRAPER EXPERIMENTAL DESIGN FOR FIVE FACTORS

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
1	2608	1564	2477	3419	10257	.419	.419	.419	.419	.419	15281.8667
2	2608	1564	2477	3419	7743	.419	.419	.419	.419	-.419	14230.5615
3	2608	1564	2477	2581	10257	.419	.419	.419	-.419	.419	14760.3715
4	2608	1564	2477	2581	7743	.419	.419	.419	-.419	-.419	13610.4415
5	2608	1564	1723	3419	10257	.419	.419	-.419	.419	.419	14860.6915
6	2608	1564	1723	3419	7743	.419	.419	-.419	.419	-.419	13702.7615
7	2608	1564	1723	2581	10257	.419	.419	-.419	-.419	.419	14240.5715
8	2608	1564	1723	2581	7743	.419	.419	-.419	-.419	-.419	12967.1524
9	2608	936	2477	3419	10257	.419	-.419	.419	.419	.419	14900.2915
10	2608	936	2477	3419	7743	.419	-.419	.419	.419	-.419	13822.3615
11	2608	936	2477	2581	10257	.419	-.419	.419	-.419	.419	14360.1715
12	2608	936	2477	2581	7743	.419	-.419	.419	-.419	-.419	13109.4324
13	2608	936	1723	3419	10257	.419	-.419	-.419	.419	.419	14452.4915
14	2608	936	1723	3419	7743	.419	-.419	-.419	.419	-.419	13216.8724
15	2608	936	1723	2581	10257	.419	-.419	-.419	-.419	.419	13832.3715
16	2608	936	1723	2581	7743	.419	-.419	-.419	-.419	-.419	12445.9124
17	2042	1564	2477	3419	10257	-.419	.419	.419	.419	.419	15065.8715
18	2042	1564	2477	3419	7743	-.419	.419	.419	.419	-.419	13902.2815
19	2042	1564	2477	2581	10257	-.419	.419	.419	-.419	.419	14445.7515
20	2042	1564	2477	2581	7743	-.419	.419	.419	-.419	-.419	13200.5124

TABLE B.17 (continued)

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
21	2042	1564	1723	3419	10257	-.419	.419	-.419	.419	.419	14538.0715
22	2042	1564	1723	3419	7743	-.419	.419	-.419	.419	-.419	13307.9524
23	2042	1564	1723	2581	10257	-.419	.419	-.419	-.419	.419	13917.9515
24	2042	1564	1723	2581	7743	-.419	.419	-.419	-.419	-.419	12356.9924
25	2042	936	2477	3419	10257	-.419	-.419	.419	.419	.419	14657.6715
26	2042	936	2477	3419	7743	-.419	-.419	.419	.419	-.419	13450.2324
27	2042	936	2477	2581	10257	-.419	-.419	.419	-.419	.419	14037.5515
28	2042	936	2477	2581	7743	-.419	-.419	.419	-.419	-.419	12679.2724
29	2042	936	1723	3419	10257	-.419	-.419	-.419	.419	.419	14129.8715
30	2042	936	1723	3419	7743	-.419	-.419	-.419	.419	-.419	12786.7124
31	2042	936	1723	2581	10257	-.419	-.419	-.419	-.419	.419	13509.7515
32	2042	936	1723	2581	7743	-.419	-.419	-.419	-.419	-.419	12015.7524
33	2998	1250	2100	3000	9000	.997	0	0	0	0	14254.5115
34	1652	1250	2100	3000	9000	-.997	0	0	0	0	13487.2915
35	2325	1998	2100	3000	9000	0	.997	0	0	0	14357.1015
36	2325	502	2100	3000	9000	0	-.997	0	0	0	13384.7015
37	2325	1250	2997	3000	9000	0	0	.997	0	0	14498.8015
38	2325	1250	1203	3000	9000	0	0	-.997	0	0	13228.1324
39	2325	1250	2100	3997	9000	0	0	0	.997	0	14608.6815
40	2325	1250	2100	2003	9000	0	0	0	-.997	0	13100.2524
41	2325	1250	2100	3000	11991	0	0	0	0	.997	15061.8431
42	2325	1250	2100	3000	6009	0	0	0	0	-.997	12080.9324
43	2325	1250	2100	3000	9000	0	0	0	0	0	13870.9015

TABLE B.10

BOX-DRAPER (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR FIVE FACTORS

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
1	2059	955	1745	2606	7818	-.394	-.394	-.394	-.394	-.394	12135.5524
2	2591	1545	1745	2606	7818	.394	.394	-.394	-.394	-.394	13029.5724
3	2591	955	2455	2606	7818	.394	-.394	.394	-.394	-.394	13164.6724
4	2059	1545	2455	2606	7818	-.394	.394	.394	-.394	-.394	13250.0524
5	2591	955	1745	3394	7818	.394	-.394	-.394	.394	-.394	13264.8324
6	2059	1545	1745	3394	7818	-.394	.394	-.394	.394	-.394	13350.2124
7	2059	955	2455	3394	7818	-.394	-.394	.394	.394	-.394	13485.3124
8	2591	1545	2455	3394	7818	.394	.394	.394	.394	-.394	14209.7015
9	2591	955	1745	2606	10182	.394	-.394	-.394	-.394	.394	13834.4315
10	2059	1545	1745	2606	10182	-.394	.394	-.394	-.394	.394	13914.6915
11	2059	955	2455	2606	10182	-.394	-.394	.394	-.394	.394	14028.1915
12	2591	1545	2455	2606	10182	.394	.394	.394	-.394	.394	14714.9315
13	2059	955	1745	3394	10182	-.394	-.394	-.394	.394	.394	14114.3115
14	2591	1545	1745	3394	10182	.394	.394	-.394	.394	.394	14801.0515
15	2591	955	2455	3394	10182	.394	-.394	.394	.394	.394	14914.5515
16	2059	1545	2455	3394	10182	-.394	.394	.394	.394	.394	14994.8115

TABLE B.18 (continued)

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
17	2957	1250	2100	3000	9000	.937	0	0	0	0	14231.1415
18	1693	1250	2100	3000	9000	-.937	0	0	0	0	13510.6615
19	2325	1953	2100	3000	9000	0	.937	0	0	0	14327.8515
20	2325	547	2100	3000	9000	0	-.937	0	0	0	13413.9515
21	2325	1250	2943	3000	9000	0	0	.937	0	0	14461.0015
22	2325	1250	1257	3000	9000	0	0	-.937	0	0	13275.6524
23	2325	1250	2100	3937	9000	0	0	0	.937	0	14564.2815
24	2325	1250	2100	2063	9000	0	0	0	-.937	0	13155.4524
25	2325	1250	2100	3000	11811	0	0	0	0	.937	15020.3710
26	2325	1250	2100	3000	6189	0	0	0	0	-.937	12190.5924
27	2325	1250	2100	3000	9000	0	0	0	0	0	13870.9015

TABLE B.19

MINIMUM POINT EXPERIMENTAL DESIGN FOR FIVE FACTORS

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
1	1650	500	1200	2000	6000	-1	-1	-1	-1	-1	9200.2424
2	3000	500	1200	2000	6000	1	-1	-1	-1	-1	10251.2424
3	1650	2000	1200	2000	6000	-1	1	-1	-1	-1	10456.7424
4	1650	500	3000	2000	6000	-1	-1	1	-1	-1	10812.2424
5	1650	500	1200	4000	6000	-1	-1	-1	1	-1	11040.2424
6	1650	500	1200	2000	12000	-1	-1	-1	-1	1	13000.6515
7	1778	642	1200	2000	6000	-.8108	-.8108	-1	-1	-1	9419.3624
8	1778	500	1370	2000	6000	-.8108	-1	-.8108	-1	-1	9453.0824
9	1778	500	1200	2189	6000	-.8108	-1	-1	-.8108	-1	9473.9624
10	1778	500	1200	2000	6568	-.8108	-1	-1	-1	-.8108	9680.6424
11	1650	642	1370	2000	6000	-1	-.8108	-.8108	-1	-1	9472.5224
12	1650	642	1200	2189	6000	-1	-.8108	-1	-.8108	-1	9493.4024
13	1650	642	1200	2000	6568	-1	-.8108	-1	-1	-.8108	9700.0824
14	1650	500	1370	2189	6000	-1	-1	-.8108	-.8108	-1	9527.1224
15	1650	500	1370	2000	6568	-1	-1	-.8108	-1	-.8108	9733.0824
16	1650	500	1200	2189	6568	-1	-1	-1	-.8108	-.8108	9754.6824
17	2686	2000	3000	4000	12000	.5355	1	1	1	1	16396.5653
18	3000	1652	3000	4000	12000	1	.5355	1	1	1	16364.7917
19	3000	2000	2582	4000	12000	1	1	.5355	1	1	16313.0837
20	3000	2000	3000	3535	12000	1	1	1	.5355	1	16276.9228
21	3000	2000	3000	4000	10606	1	1	1	1	.5355	16197.7212

TABLE B.20

KOSHAL EXPERIMENTAL DESIGN FOR FIVE FACTORS

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
1	2100	1000	1800	2667	8000	-.33	-.33	-.33	-.33	-.33	12426.8824
2	2775	1000	1800	2667	8000	.67	-.33	-.33	-.33	-.33	12937.1324
3	2100	1750	1800	2667	8000	-.33	.67	-.33	-.33	-.33	13049.3824
4	2100	1000	2700	2667	8000	-.33	-.33	.67	-.33	-.33	13218.8824
5	2100	1000	1800	3667	8000	-.33	-.33	-.33	.67	-.33	13346.8824
6	2100	1000	1800	2667	11000	-.33	-.33	-.33	-.33	.67	14043.7315
7	3450	1000	1800	2667	8000	1.67	-.33	-.33	-.33	-.33	13433.2315
8	2100	2500	1800	2667	8000	-.33	1.67	-.33	-.33	-.33	13634.7315
9	2100	1000	3600	2667	8000	-.33	-.33	1.67	-.33	-.33	13919.7315
10	2100	1000	1800	4667	8000	-.33	-.33	-.33	1.67	-.33	14139.7315
11	2100	1000	1800	2667	14000	-.33	-.33	-.33	-.33	1.67	15031.9435
12	2775	1750	1800	2667	8000	.67	.67	-.33	-.33	-.33	13535.9815
13	2775	1000	2700	2667	8000	.67	-.33	.67	-.33	-.33	13678.4815
14	2775	1000	1800	3667	8000	.67	-.33	-.33	.67	-.33	13788.4815
15	2775	1000	1800	2667	11000	.67	-.33	-.33	-.33	.67	14428.4815
16	2100	1750	2700	2667	8000	-.33	.67	.67	-.33	-.33	13777.2315
17	2100	1750	1800	3667	8000	-.33	.67	-.33	.67	-.33	13887.2315
18	2100	1750	1800	2667	11000	-.33	.67	-.33	-.33	.67	14531.2315
19	2100	1000	2700	3667	8000	-.33	-.33	.67	.67	-.33	14029.7315
20	2100	1000	2700	2667	11000	-.33	-.33	.67	-.33	.67	14673.7315
21	2100	1000	1800	3667	11000	-.33	-.33	-.33	.67	.67	14783.7315

TABLE B.21

CENTRAL COMPOSITE ROTATABLE (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR FIVE FACTORS

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
1	1650	500	1200	2000	6000	-1	-1	-1	-1	-1	9200.2424
2	3000	2000	1200	2000	6000	1	1	-1	-1	-1	11496.2424
3	3000	500	3000	2000	6000	1	-1	1	-1	-1	11838.2424
4	1650	2000	3000	2000	6000	-1	1	1	-1	-1	12057.2424
5	3000	500	1200	4000	6000	1	-1	-1	1	-1	12091.2424
6	1650	2000	1200	4000	6000	-1	1	-1	1	-1	12296.7424
7	1650	500	3000	4000	6000	-1	-1	1	1	-1	12652.2424
8	3000	2000	3000	4000	6000	1	1	1	1	-1	14718.1515
9	3000	500	1200	2000	12000	1	-1	-1	-1	1	13778.1515
10	1650	2000	1200	2000	12000	-1	1	-1	-1	1	13983.6515
11	1650	500	3000	2000	12000	-1	-1	1	-1	1	14268.6515
12	3000	2000	3000	2000	12000	1	1	1	-1	1	15521.9467
13	1650	500	1200	4000	12000	-1	-1	-1	1	1	14488.6515
14	3000	2000	1200	4000	12000	1	1	-1	1	1	15694.2388
15	3000	500	3000	4000	12000	1	-1	1	1	1	15904.3588
16	1650	2000	3000	4000	12000	-1	1	1	1	1	16058.9188

TABLE B.21 (continued)

	NON-CODED					CODED					
RUN #	M1	M2	B1	B2	S1	M1	M2	B1	B2	S1	RESPONSE
17	3675	1250	2100	3000	9000	2	0	0	0	0	14640.4015
18	975	1250	2100	3000	9000	-2	0	0	0	0	12999.7424
19	2325	2750	2100	3000	9000	0	2	0	0	0	14845.[illegible]15
20	2325	0	2100	3000	9000	0	-2	0	0	0	12979.9924
21	2325	1250	3900	3000	9000	0	0	2	0	0	15130.9015
22	2325	1250	300	3000	9000	0	0	-2	0	0	12433.4924
23	2325	1250	2100	5000	9000	0	0	0	2	0	15350.9015
24	2325	1250	2100	1000	9000	0	0	0	-2	0	12177.4924
25	2325	1250	2100	3000	15000	0	0	0	0	2	15747.0788
26	2325	1250	2100	3000	3000	0	0	0	0	-2	10076.7424
27	2325	1250	2100	3000	9000	0	0	0	0	0	13870.9015

TABLE B.22

BOX-BEHNKEN EXPERIMENTAL DESIGN FOR SIX FACTORS

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
1	2325	2000	700	2100	4000	12000	0	1	0	0	1	1	16271.1000
2	2325	2000	700	2100	4000	6000	0	1	0	0	1	-1	14203.5682
3	2325	2000	700	2100	2000	12000	0	1	0	0	-1	1	15246.5500
4	2325	2000	700	2100	2000	6000	0	1	0	0	-1	-1	12417.1591
5	2325	500	700	2100	4000	12000	0	-1	0	0	1	1	15644.7500
6	2325	500	700	2100	4000	6000	0	-1	0	0	1	-1	11172.1591
7	2325	500	700	2100	2000	12000	0	-1	0	0	-1	1	14531.0682
8	2325	500	700	2100	2000	6000	0	-1	0	0	-1	-1	13012.1591
9	3000	1250	1000	2100	3000	12000	1	0	1	0	0	1	15817.5400
10	3000	1250	1000	2100	3000	6000	1	0	1	0	0	-1	13489.4091
11	3000	1250	400	2100	3000	12000	1	0	-1	0	0	1	15541.3000
12	3000	1250	400	2100	3000	6000	1	0	-1	0	0	-1	12967.4091
13	1650	1250	1000	2100	3000	12000	-1	0	1	0	0	1	15358.0000
14	1650	1250	1000	2100	3000	6000	-1	0	1	0	0	-1	12455.9091
15	1650	1250	400	2100	3000	12000	-1	0	-1	0	0	1	11933.9091
16	1650	1250	400	2100	3000	6000	-1	0	-1	0	0	-1	15058.0000
17	3000	2000	400	3000	3000	9000	1	1	0	1	0	0	15715.0000
18	3000	2000	400	1200	3000	9000	1	1	0	-1	0	0	14620.8182
19	3000	500	400	3000	3000	9000	1	-1	0	1	0	0	14905.8182
20	3000	500	400	1200	3000	9000	1	-1	0	-1	0	0	13645.8182
21	1650	2000	400	3000	3000	9000	-1	1	0	1	0	0	15111.3182
22	1650	2000	400	1200	3000	9000	-1	1	0	-1	0	0	13851.3182
23	1650	500	400	3000	3000	9000	-1	-1	0	1	0	0	14136.3182
24	1650	500	400	1200	3000	9000	-1	-1	0	-1	0	0	12736.4091
25	2325	2000	1000	2100	4000	9000	0	1	1	0	1	0	15699.5000
26	2325	2000	1000	2100	2000	9000	0	1	1	0	-1	0	14333.0682
27	2325	2000	400	2100	4000	9000	0	1	-1	0	1	0	15399.0682
28	2325	2000	400	2100	2000	9000	0	1	-1	0	-1	0	13919.0682
29	2325	500	1000	2100	4000	9000	0	-1	1	0	1	0	14838.0682
30	2325	500	1000	2100	2000	9000	0	-1	1	0	-1	0	13358.0682
31	2325	500	400	2100	4000	9000	0	-1	-1	0	1	0	14424.0682
32	2325	500	400	2100	2000	9000	0	-1	-1	0	-1	0	12853.6591

TABLE B.22 (continued)

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
33	2325	1250	1000	3000	3000	12000	0	0	1	1	0	1	16011.1300
34	2325	1250	1000	3000	3000	6000	0	0	1	1	0	-1	13768.4091
35	2325	1250	1000	1200	3000	12000	0	0	1	-1	0	1	15154.0000
36	2325	1250	1000	1200	3000	6000	0	0	1	-1	0	-1	12174.6591
37	2325	1250	400	3000	3000	12000	0	0	-1	1	0	1	15734.8900
38	2325	1250	400	3000	3000	6000	0	0	-1	1	0	-1	13246.4091
39	2325	1250	400	1200	3000	12000	0	0	-1	-1	0	1	14855.5000
40	2325	1250	400	1200	3000	6000	0	0	-1	-1	0	-1	11652.6591
41	3000	1250	700	3000	4000	9000	1	0	0	1	1	0	15920.0000
42	3000	1250	700	3000	2000	9000	1	0	0	1	-1	0	14653.3182
43	3000	1250	700	1200	4000	9000	1	0	0	-1	1	0	14873.3182
44	3000	1250	700	1200	2000	9000	1	0	0	-1	-1	0	13393.3182
45	1650	1250	700	3000	4000	9000	-1	0	0	1	1	0	15363.8182
46	1650	1250	700	3000	2000	9000	-1	0	0	1	-1	0	13883.8182
47	1650	1250	700	1200	4000	9000	-1	0	0	-1	1	0	14103.8182
48	1650	1250	700	1200	2000	9000	-1	0	0	-1	-1	0	12438.9091
49	2325	1250	700	2100	3000	9000	0	0	0	0	0	0	14378.5682

TABLE B.23

BOX-DRAPER (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR SIX FACTORS

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
1	2042	936	574	1723	2581	10257	-.419	-.419	-.419	-.419	-.419	.419	13930.5553
2	2042	936	574	1723	3419	7743	-.419	-.419	-.419	-.419	.419	-.419	13316.8614
3	2042	936	574	2477	2581	7743	-.419	-.419	-.419	.419	-.419	-.419	13209.5974
4	2049	936	574	2477	3419	10257	-.419	-.419	-.419	.419	.419	.419	15078.6153
5	2042	936	826	1723	2581	7743	-.419	-.419	.419	-.419	-.419	-.419	12764.6194
6	2042	936	826	1723	3419	10257	-.419	-.419	.419	-.419	.419	.419	14724.1413
7	2042	936	826	2477	2581	10257	-.419	-.419	.419	.419	-.419	.419	14631.9613
8	2042	936	826	2477	3419	7743	-.419	-.419	.419	.419	.419	-.419	14088.4931
9	2042	1564	574	1723	2581	7743	-.419	.419	-.419	-.419	-.419	-.419	13067.5564
10	2042	1564	574	1723	3419	10257	-.419	.419	-.419	-.419	.419	.419	14959.2003
11	2042	1564	574	2477	2581	10257	-.419	.419	-.419	.419	-.419	.419	14867.0203
12	2042	1564	574	2477	3419	7743	-.419	.419	-.419	.419	.419	-.419	14323.5521
13	2042	1564	826	1723	2581	10257	-.419	.419	.419	-.419	-.419	.419	14512.5463
14	2042	1564	826	1723	3419	7743	-.419	.419	.419	-.419	.419	-.419	13969.0781
15	2042	1564	826	2477	2581	7743	-.419	.419	.419	.419	-.419	-.419	13876.8981
16	2042	1564	826	2477	3419	10257	-.419	.419	.419	.419	.419	.419	15516.5944
17	2608	936	574	1723	2581	7743	.419	-.419	-.419	-.419	-.419	-.419	12975.7954
18	2608	936	574	1723	3419	10257	.419	-.419	-.419	-.419	.419	.419	14873.0958
19	2608	936	574	2477	2581	10257	.419	-.419	-.419	.419	-.419	.419	14780.9158
20	2608	936	574	2477	3419	7743	.419	-.419	-.419	.419	.419	-.419	14243.1041
21	2608	936	826	1723	2581	10257	.419	-.419	.419	-.419	-.419	.419	14426.4418
22	2608	936	826	1723	3419	7743	.419	-.419	.419	-.419	.419	-.419	13888.6301
23	2608	936	826	2477	2581	7743	.419	-.419	.419	.419	-.419	-.419	13796.4501
24	2608	936	826	2477	3419	10257	.419	-.419	.419	.419	.419	.419	15442.4314
25	2608	1564	574	1723	2581	10257	.419	.419	-.419	-.419	-.419	.419	14661.5008
26	2608	1564	574	1723	3419	7743	.419	.419	-.419	-.419	.419	-.419	14123.6891
27	2608	1564	574	2477	2581	7743	.419	.419	-.419	.419	-.419	-.419	14031.5091
28	2608	1564	574	2477	3419	10257	.419	.419	-.419	.419	.419	.419	15605.8414
29	2608	1564	826	1723	2581	7743	.419	.419	.419	-.419	-.419	-.419	13677.0351
30	2608	1564	826	1723	3419	10257	.419	.419	.419	-.419	.419	.419	15346.8994
31	2608	1564	826	2477	2581	10257	.419	.419	.419	.419	-.419	.419	15270.6414
32	2608	1564	826	2477	3419	7743	.419	.419	.419	.419	.419	-.419	14825.0951

TABLE B.23 (continued)

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
33	2998	1250	700	2100	3000	9000	.997	0	0	0	0	0	14762.1668
34	1652	1250	700	2100	3000	9000	-.997	0	0	0	0	0	13994.9753
35	2325	1998	700	2100	3000	9000	0	.997	0	0	0	0	14864.6057
36	2325	502	700	2100	3000	9000	0	-.997	0	0	0	0	13892.5307
37	2325	1250	999	2100	3000	9000	0	0	.997	0	0	0	14584.9472
38	2325	1250	401	2100	3000	9000	0	0	-.997	0	0	0	14172.1892
39	2325	1250	700	2997	3000	9000	0	0	0	.997	0	0	15006.6782
40	2325	1250	700	1203	3000	9000	0	0	0	-.997	0	0	13750.4582
41	2325	1250	700	2100	3997	9000	0	0	0	0	.997	0	15116.3482
42	2325	1250	700	2100	2003	9000	0	0	0	0	-.997	0	13640.7882
43	2325	1250	700	2100	3000	11991	0	0	0	0	0	.997	15447.5764
44	2325	1250	700	2100	3000	6009	0	0	0	0	0	-.997	12720.5991
45	2325	1250	700	2100	3000	9000	0	0	0	0	0	0	14378.5682

TABLE B.24

HYBRID (628A) EXPERIMENTAL DESIGN FOR SIX FACTORS

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
1	2325	1250	700	2100	3000	16103	0	0	0	0	0	2.3677	16380.4422
2	2325	1250	700	2100	3000	3567	0	0	0	0	0	-1.8110	11096.2991
3	1650	500	400	1200	2000	10829	-1	-1	-1	-1	-1	.6096	12725.8411
4	3000	2000	400	1200	2000	10829	1	1	-1	-1	-1	.6096	14515.0662
5	3000	500	1000	1200	2000	10829	1	-1	1	-1	-1	.6096	13954.0662
6	1650	2000	1000	1200	2000	10829	-1	1	1	-1	-1	.6096	14159.5662
7	3000	500	400	3000	2000	10829	1	-1	-1	1	-1	.6096	14000.0662
8	1650	2000	400	3000	2000	10829	-1	1	-1	1	-1	.6096	14995.7760
9	1650	1250	1000	3000	2000	10829	-1	-1	1	1	-1	.6096	14444.5662
10	3000	2000	1000	3000	2000	10829	1	1	1	1	-1	.6096	15775.9555
11	3000	500	400	1200	4000	10829	1	-1	-1	-1	1	.6096	15000.7760
12	1650	2000	400	1200	4000	10829	-1	1	-1	-1	1	.6096	15177.7760
13	1650	500	1000	1200	4000	10829	-1	-1	1	-1	1	.6096	14664.5662
14	3000	2000	1000	1200	4000	10829	1	1	1	-1	1	.6096	15950.0355
15	1650	500	400	3000	4000	10829	-1	-1	-1	1	1	.6096	15405.7760
16	3000	2000	400	3000	4000	10829	1	1	-1	1	1	.6096	16509.2275
17	3000	500	1000	3000	4000	10829	1	-1	1	1	1	.6096	16166.1555
18	1650	2000	1000	3000	4000	10829	-1	1	1	1	1	.6096	16327.4275

TABLE B.24 (continued)

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
19	3793	1250	700	2100	3000	5907	2.1749	0	0	0	0	-1.031	13770.6847
20	857	1250	700	2100	3000	5907	-2.1749	0	0	0	0	-1.031	11519.0122
21	2325	2881	700	2100	3000	5907	0	2.1749	0	0	0	-1.031	13991.6952
22	2325	0	700	2100	3000	5907	0	-2.1749	0	0	0	-1.031	11614.0991
23	2325	1250	1352	2100	3000	5907	0	0	2.1749	0	0	-1.031	13220.9280
24	2325	1250	48	2100	3000	5907	0	0	-2.1749	0	0	-1.031	12085.6302
25	2325	1250	700	4057	3000	5907	0	0	0	2.1749	0	-1.031	14303.2952
26	2325	1250	700	143	3000	5907	0	0	0	-2.1749	0	-1.031	10902.4301
27	2325	1250	700	2100	5175	5907	0	0	0	0	2.1749	-1.031	14540.8542
28	2325	1250	700	2100	825	5907	0	0	0	0	-2.1749	-1.031	10652.3711

TABLE B.25

HYBRID (628B) EXPERIMENTAL DESIGN FOR SIX FACTORS

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
1	2325	1250	700	2100	3000	15928	0	0	0	0	0	2.3094	16343.6433
2	2325	1250	700	2100	3000	9000	0	0	0	0	0	0	14378.5682
3	1650	500	400	1200	2000	10732	-1	-1	-1	-1	-1	.5773	12663.8251
4	3000	2000	400	1200	2000	10732	1	1	-1	-1	-1	.5773	14470.4922
5	3000	500	1000	1200	2000	10732	1	-1	1	-1	-1	.5773	13909.4922
6	1650	2000	1000	1200	2000	10732	-1	1	1	-1	-1	.5773	14114.9922
7	3000	500	400	3000	2000	10732	1	-1	-1	1	-1	.5773	14755.4922
8	1650	2000	400	3000	2000	10732	-1	1	-1	1	-1	.5773	14960.9922
9	1650	1250	1000	3000	2000	10732	-1	-1	1	1	-1	.5773	14399.9922
10	3000	2000	1000	3000	2000	10732	1	1	1	1	-1	.5773	15753.6298
11	3000	500	400	1200	4000	10732	1	-1	-1	-1	1	.5773	14974.6130
12	1650	2000	400	1200	4000	10732	-1	1	-1	-1	1	.5773	15151.6130
13	1650	500	1000	1200	4000	10732	-1	-1	1	-1	1	.5773	14619.9922
14	3000	2000	1000	1200	4000	10732	1	1	1	-1	1	.5773	15927.7098
15	1650	500	400	3000	4000	10732	-1	-1	-1	1	1	.5773	15379.6130
16	3000	2000	400	3000	4000	10732	1	1	-1	1	1	.5773	16487.8708
17	3000	500	1000	3000	4000	10732	1	-1	1	1	1	.5773	16143.8298
18	1650	2000	1000	3000	4000	10732	-1	1	1	1	1	.5773	16306.0708

TABLE B.25 (continued)

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
19	3675	1250	700	2100	3000	5536	2.0	0	0	0	0	-1.1547	13439.7441
20	975	1250	700	2100	3000	5536	-2.0	0	0	0	0	-1.1547	11362.4621
21	2325	2750	700	2100	3000	5536	0	2.0	0	0	0	-1.1547	13653.3531
22	2325	0	700	2100	3000	5536	0	-2.0	0	0	0	-1.1547	11361.5091
23	2325	1250	1300	2100	3000	5536	0	0	2.0	0	0	-1.1547	12930.3531
24	2325	1250	100	2100	3000	5536	0	0	-2.0	0	0	-1.1547	11886.3531
25	2325	1250	700	3900	3000	5536	0	0	0	2.0	0	-1.1547	13997.7441
26	2325	1250	700	300	3000	5536	0	0	0	-2.0	0	-1.1547	10795.4621
27	2325	1250	700	2100	5000	5536	0	0	0	0	2.0	-1.1547	14233.3002
28	2325	1250	700	2100	1000	5536	0	0	0	0	-2.0	-1.1547	10568.3531

TABLE B.26

MINIMUM POINT EXPERIMENTAL DESIGN FOR SIX FACTORS

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
1	2742	1714	400	1200	2000	6000	.6183	.6183	-1	-1	-1	-1	11438.9145
2	2742	500	885	1200	2000	6000	.6183	-1	.6183	-1	-1	-1	10853.3184
3	2742	500	400	2656	2000	6000	.6183	-1	-1	.6183	-1	-1	11718.7887
4	2742	500	400	1200	3618	6000	.6183	-1	-1	-1	.6183	-1	11919.7781
5	2742	500	400	1200	2000	10855	.6183	-1	-1	-1	-1	.6183	13405.2117
6	1650	1714	885	1200	2000	6000	-1	.6183	.6183	-1	-1	-1	11020.1813
7	1650	1714	400	2656	2000	6000	-1	.6183	-1	.6183	-1	-1	11894.0633
8	1650	1714	400	1200	3618	6000	-1	.6183	-1	-1	.6183	-1	12006.6410
9	1650	1714	400	1200	2000	10855	-1	.6183	-1	-1	-1	.6183	13571.4967
10	1650	500	885	2656	2000	6000	-1	-1	.6183	.6183	-1	-1	11309.0437
11	1650	500	885	1200	3618	6000	-1	-1	.6183	-1	.6183	-1	11490.1214
12	1650	500	885	1200	2000	10885	-1	-1	.6183	-1	-1	.6183	13117.5603
13	1650	500	400	2656	3618	6000	-1	-1	-1	.6183	.6183	-1	12375.5034
14	1650	500	400	2656	2000	10885	-1	-1	-1	.6183	-1	.6183	13802.1012
15	1650	500	400	1200	3618	10885	-1	-1	-1	-1	.6183	.6183	13980.1142
16	1727	2000	1000	3000	4000	12000	-.8854	1	1	1	1	1	16611.1197
17	3000	586	1000	3000	4000	12000	1	-.8854	1	1	1	1	16465.4144
18	3000	2000	434	3000	4000	12000	1	1	-.8854	1	1	1	16782.5009
19	3000	2000	1000	1303	4000	12000	1	1	1	-.8854	1	1	16267.3343
20	3000	2000	1000	3000	2115	12000	1	1	1	1	-.8854	1	16104.2918

TABLE B.26 (continued)

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
21	3000	2000	1000	3000	4000	6344	1	1	1	1	1	-.8854	15594.4042
22	3000	500	400	1200	2000	6000	1	-1	-1	-1	-1	-1	10629.9091
23	1650	2000	400	1200	2000	6000	-1	1	-1	-1	-1	-1	10835.4091
24	1650	500	1000	1200	2000	6000	-1	-1	1	-1	-1	-1	10100.9091
25	1650	500	400	3000	2000	6000	-1	-1	-1	1	-1	-1	11190.9091
26	1650	500	400	1200	4000	6000	-1	-1	-1	-1	1	-1	11418.9091
27	1650	500	400	1200	2000	12000	-1	-1	-1	-1	-1	1	13309.3182
28	1650	500	400	1200	2000	6000	-1	-1	-1	-1	-1	-1	9578.9091

TABLE B.27

KOSHAL EXPERIMENTAL DESIGN FOR SIX FACTORS

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
1	2807	1786	614	1843	2714	8143	.7143	.7143	-.2857	-.2857	-.2857	-.2857	14156.8033
2	2807	1036	914	1843	2714	8143	.7143	-.2857	.7143	-.2857	-.2857	-.2857	13876.3033
3	2807	1036	614	2743	2714	8143	.7143	-.2857	-.2857	.7143	-.2857	-.2857	14299.3033
4	2807	1036	614	1843	3714	8143	.7143	-.2857	-.2857	-.2857	.7143	-.2857	14409.3033
5	2807	1036	614	1843	2714	11143	.7143	-.2857	-.2857	-.2857	-.2857	.7143	14986.6095
6	2132	1786	914	1843	2714	8143	-.2857	.7143	.7143	-.2857	-.2857	-.2857	13976.8038
7	2132	1786	614	2743	2714	8143	-.2857	.7143	-.2857	.7143	-.2857	-.2857	14399.8038
8	2132	1786	614	1843	3714	8143	-.2857	.7143	-.2857	-.2857	.7143	-.2857	14509.8038
9	2132	1786	614	1843	2714	11143	-.2857	.7143	-.2857	-.2857	-.2857	.7143	15075.1095
10	2132	1036	914	2743	2714	8143	-.2857	-.2857	.7143	.7143	-.2857	-.2857	14119.3038
11	2132	1036	914	1843	3714	8143	-.2857	-.2857	.7143	-.2857	.7143	-.2857	14229.3038
12	2132	1036	914	1843	2714	11143	-.2857	-.2857	.7143	-.2857	-.2857	.7143	14871.5533
13	2132	1036	614	2743	3714	8143	-.2857	-.2857	-.2857	.7143	.7143	-.2857	14652.3038
14	2132	1036	614	2743	2714	11143	-.2857	-.2857	-.2857	.7143	-.2857	.7143	15189.1095
15	2132	1036	614	2743	3714	11143	-.2857	-.2857	-.2857	-.2857	.7143	.7143	15280.1095
16	3482	1036	614	1843	2714	8143	1.7143	-.2857	-.2857	-.2857	-.2857	-.2857	14054.0533
17	2132	2536	614	1843	2714	8143	-.2857	1.7143	-.2857	-.2857	-.2857	-.2857	14257.3038
18	2132	1036	1214	1843	2714	8143	-.2857	-.2857	1.7143	-.2857	-.2857	-.2857	13696.3038
19	2132	1036	614	3643	2714	8143	-.2857	-.2857	-.2857	1.7143	-.2857	-.2857	14542.3038
20	2132	1036	614	1843	4714	8143	-.2857	-.2857	-.2857	-.2857	1.7143	-.2857	14762.3038

TABLE B.27 (continued)

RUN #	NON-CODED M1	NON-CODED M2	NON-CODED M3	NON-CODED B1	NON-CODED B2	NON-CODED S1	CODED M1	CODED M2	CODED M3	CODED B1	CODED B2	CODED S1	RESPONSE
21	2132	1036	614	1843	2714	14143	-.2857	-.2857	-.2857	-.2857	-.2857	1.7143	15481.4128
22	2807	1036	614	1843	2714	8143	.7143	-.2857	-.2857	-.2857	-.2857	-.2857	13669.3033
23	2132	1786	614	1843	2714	8143	-.2857	.7143	-.2857	-.2857	-.2857	-.2857	13769.8038
24	2132	1036	914	1843	2714	8143	-.2857	-.2857	.7143	-.2857	-.2857	-.2857	13481.1979
25	2132	1036	614	2743	2714	8143	-.2857	-.2857	-.2857	.7143	-.2857	-.2857	13912.3038
26	2132	1036	614	1843	3714	8143	-.2857	-.2857	-.2857	-.2857	.7143	-.2857	14022.3038
27	2132	1036	614	1843	2714	11143	-.2857	-.2857	-.2857	-.2857	-.2857	.7143	14664.5533
28	2132	1036	614	1843	2714	8143	-.2857	-.2857	-.2857	-.2857	-.2857	-.2857	13220.1979

TABLE B.28

CENTRAL COMPOSITE ROTATABLE (HALF-REPLICATE) EXPERIMENTAL DESIGN FOR SIX FACTORS

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
1	1650	500	400	1200	2000	12000	-1	-1	-1	-1	-1	1	13309.3182
2	1650	500	400	1200	4000	6000	-1	-1	-1	-1	1	-1	11418.9091
3	1650	500	400	3000	2000	6000	-1	-1	-1	1	-1	-1	11190.9091
4	1650	500	400	3000	4000	12000	-1	-1	-1	1	1	1	15700.2200
5	1650	500	1000	1200	2000	6000	-1	-1	1	-1	-1	-1	10100.9091
6	1650	500	1000	1200	4000	12000	-1	-1	1	-1	1	1	15104.0000
7	1650	500	1000	3000	2000	12000	-1	-1	1	1	-1	1	14922.0000
8	1650	500	1000	3000	4000	6000	-1	-1	1	1	1	-1	13552.9091
9	1650	2000	400	1200	2000	6000	-1	1	-1	-1	-1	-1	10835.4091
10	1650	2000	400	1200	4000	12000	-1	1	-1	-1	1	1	15484.1000
11	1650	2000	400	3000	2000	12000	-1	1	-1	1	-1	1	15310.0200
12	1650	2000	400	3000	4000	6000	-1	1	-1	1	1	-1	14235.8182
13	1650	2000	1000	1200	2000	12000	-1	1	1	-1	-1	1	14691.0000
14	1650	2000	1000	1200	4000	6000	-1	1	1	-1	1	-1	13197.4091
15	1650	2000	1000	3000	2000	6000	-1	1	1	1	-1	-1	12957.9091
16	1650	2000	1000	3000	4000	12000	-1	1	1	1	1	1	16585.5600
17	3000	500	400	1200	2000	6000	1	-1	-1	-1	-1	-1	10629.9091
18	3000	500	400	1200	4000	12000	1	-1	-1	-1	1	1	15313.0400
19	3000	500	400	3000	2000	12000	1	-1	-1	1	-1	1	15135.0000
20	3000	500	400	3000	4000	6000	1	-1	-1	1	1	-1	14043.8182
21	3000	500	1000	1200	2000	12000	1	-1	1	-1	-1	1	14492.8182
22	3000	500	1000	1200	4000	6000	1	-1	1	-1	1	-1	12991.9091
23	3000	500	1000	3000	2000	6000	1	-1	1	1	-1	-1	12738.9091
24	3000	500	1000	3000	4000	12000	1	-1	1	1	1	1	16431.0000
25	3000	2000	400	1200	2000	12000	1	1	-1	-1	-1	1	14889.0000
26	3000	2000	400	1200	4000	6000	1	1	-1	-1	1	-1	13714.9091
27	3000	2000	400	3000	2000	6000	1	1	-1	1	-1	-1	13461.9091
28	3000	2000	400	3000	4000	12000	1	1	-1	1	1	1	16767.3600
29	3000	2000	1000	1200	2000	6000	1	1	1	-1	-1	-1	12396.9091
30	3000	2000	1000	1200	4000	12000	1	1	1	-1	1	1	16219.8800
31	3000	2000	1000	3000	2000	12000	1	1	1	1	-1	1	16045.8000
32	3000	2000	1000	3000	4000	6000	1	1	1	1	1	-1	15432.8182

TABLE B.28 (continued)

	NON-CODED						CODED						
RUN #	M1	M2	M3	B1	B2	S1	M1	M2	M3	B1	B2	S1	RESPONSE
33	3930	1250	700	2100	3000	9000	2.378	0	0	0	0	0	15264.4570
34	720	1250	700	2100	3000	9000	-2.378	0	0	0	0	0	13445.4951
35	2325	3034	700	2100	3000	9000	0	2.378	0	0	0	0	15474.9100
36	2325	0	700	2100	3000	9000	0	-2.378	0	0	0	0	13566.0682
37	2325	1250	1413	2100	3000	9000	0	0	2.378	0	0	0	14870.8142
38	2325	1250	0	2100	3000	9000	0	0	-2.378	0	0	0	13895.5682
39	2325	1250	700	4240	3000	9000	0	0	0	2.378	0	0	15746.0020
40	2325	1250	700	0	3000	9000	0	0	0	-2.378	0	0	12809.1591
41	2325	1250	700	2100	5378	9000	0	0	0	0	2.378	0	15962.4000
42	2325	1250	700	2100	622	9000	0	0	0	0	-2.378	0	12469.3991
43	2325	1250	700	2100	3000	16134	0	0	0	0	0	2.378	16386.9436
44	2325	1250	700	2100	3000	1866	0	0	0	0	0	-2.378	9953.7664
45	2325	1250	700	2100	3000	9000	0	0	0	0	0	0	14378.5682

Appendix C: Linear Regression Results

This section contains the results of the stepwise linear regression run of a sample input for the three factor problem. The experimental design which was used to complete this sample run is the Box-Draper bias minimizing design for three factors.

BMDP2R - STEPWISE REGRESSION
BMDP STATISTICAL SOFTWARE, INC.
1964 WESTWOOD BLVD. SUITE 202
LOS ANGELES, CA, USA 90025
(213) 475-5700
PROGRAM REVISED OCTOBER 1983
MANUAL REVISED -- 1983

PROGRAM CONTROL INFORMATION

```
/problem        title is 'sample for three factor problem'.

/input          variables are 4.
                format is free.
                file is bd3a.

/variable       names are M1,B1,S1,DE,
                          M1sqd,B1sqd,S1sqd,
                          M1B1,M1S1,B1S1.

                add = 6.

/transform      M1sqd = M1 * M1.
                B1sqd = B1 * B1.
                S1sqd = S1 * S1.
                M1B1 = M1 * B1.
                M1S1 = M1 * S1.
                B1S1 = B1 * S1.

/regress        depend is DE.
                independent are M1,B1,S1,
                                M1sqd,B1sqd,S1sqd,
                                M1B1,M1S1,B1S1.
                tol = .001.

/print
                covariance.
                correlation.
                rreg.
/end.
```

```
PROBLEM TITLE IS
sample input for 3 factor problem

NUMBER OF VARIABLES TO READ IN. . . . . . . . .        4
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. .        6
TOTAL NUMBER OF VARIABLES . . . . . . . . . . .       10
NUMBER OF CASES TO READ IN. . . . . . . . . . . TO END
CASE  LABELING VARIABLES . . . . . . . . . . . . . . .
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. . NEITHER
BLANKS ARE. . . . . . . . . . . . . . . . . . . MISSING
INPUT FILE. . . . . . . . . UNIT    7 . . . . . .bd3a

REWIND INPUT UNIT PRIOR TO READING. . DATA. . .      YES
NUMBER OF WORDS OF DYNAMIC STORAGE. . . . . . .    25598

VARIABLES TO BE USED
     1 M1          2 B1          3 S1          4 DE          5 M1sqd
     6 B1sqd       7 S1sqd       8 M1B1        9 M1S1       10 B1S1

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS   80 CHARACTERS.



REGRESSION INTERCEPT. . . . . . . . . . . . . .NON ZERO
WEIGHT VARIABLE . . . . . . . . . . . . . . . .
PRINT COVARIANCE MATRIX . . . . . . . . . . . .      YES
PRINT CORRELATION MATRIX. . . . . . . . . . . .      YES
PRINT ANOVA AT EACH STEP. . . . . . . . . . . .      YES
PRINT STEP OUTPUT . . . . . . . . . . . . . . .      YES
PRINT REGRESSION COEFFICIENT SUMMARY TABLE. . .      YES
PRINT PARTIAL CORRELATION SUMMARY TABLE . . . .       NO
PRINT F-RATIO SUMMARY TABLE . . . . . . . . . .       NO
PRINT SUMMARY TABLE . . . . . . . . . . . . . .      YES
PRINT RESIDUALS AND DATA. . . . . . . . . . . .       NO
PRINT CORRELATION OF REGRESSION COEFFICIENTS. .      YES
PRINT NORMAL PROBABILITY PLOT . . . . . . . . .       NO
PRINT DETRENDED NORMAL PROBABILITY PLOT . . . .       NO
PRINT DIAGNOSTIC PLOTS. . . . . . . . . . . . .       NO
PRINT PLOTS FOR XVAR AND YVAR . . . . . . . . .       NO
PRINT PLOTS AT EACH STEP. . . . . . . . . . . .       NO
PRINT PLOTS AND DATA. . . . . . . . . . . . . .       NO
PRINT PLOTS WITH STATISTICS . . . . . . . . . .       NO

NUMBER OF CASES READ. . . . . . . . . . . . . .       15
```

COVARIANCE MATRIX

		M1 1	B1 2	S1 3	M1sqd 5	B1sqd 6	S1sqd 7	M1B1 8	M1S1 9	B1S1 10
M1	1	.7434								
B1	2	.4257e-08	.7434							
S1	3	.0000e+00	.0000e+00	.7434						
M1sqd	5	-.3692e-08	.0000e+00	.8515e-08	.4796					
B1sqd	6	-.3362e-08	.0000e+00	.0000e+00	-.1838	.4796				
S1sqd	7	-.3362e-08	-.4257e-08	.0000e+00	-.1838	-.1838	.4796			
M1B1	8	.8515e-08	.2554e-07	.0000e+00	.2909e-09	.2909e-09	.2909e-09	.3319		
M1S1	9	.4257e-08	.0000e+00	.1277e-07	.0000e+00	.0000e+00	.0000e+00	-.1277e-07	.3319	
B1S1	10	.4257e-08	.8515e-08	.1277e-07	.0000e+00	.0000e+00	.0000e+00	.1277e-07	.0000e+00	.3319

CORRELATION MATRIX

		M1 1	B1 2	S1 3	M1sqd 5	B1sqd 6	S1sqd 7	M1B1 8	M1S1 9	B1S1 10
M1	1	1.0000								
B1	2	.0000	1.0000							
S1	3	.0000	.0000	1.0000						
M1sqd	5	.0000	.0000	.0000	1.0000					
B1sqd	6	.0000	.0000	.0000	-.3833	1.0000				
S1sqd	7	.0000	.0000	.0000	-.3833	-.3833	1.0000			
M1B1	8	.0000	.0000	.0000	.0000	.0000	.0000	1.0000		
M1S1	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	1.0000	
B1S1	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	1.0000

STEPPING ALGORITHM. . . . . .F
MAXIMUM NUMBER OF STEPS 20
DEPENDENT VARIABLE. . . . . 4 DE
MINIMUM ACCEPTABLE F TO ENTER 4.000, 4.000
MAXIMUM ACCEPTABLE F TO REMOVE. 3.900, 3.900
MINIMUM ACCEPTABLE TOLERANCE. 00100
SUBSCRIPTS OF THE INDEPENDENT VARIABLES 1 2 3 5 6
7 8 9 10

STEP NO. 0

STD. ERROR OF EST. 1138.3025

ANALYSIS OF VARIANCE

	SUM OF SQUARES	DF	MEAN SQUARE
RESIDUAL	18140256.	14	1295733.

VARIABLES IN EQUATION FOR DE

VARIABLE	COEFFICIENT	STD. ERROR OF COEFF	STD REG COEFF	TOLERANCE	F TO REMOVE	LEVEL
(Y-INTERCEPT	9161.82715)					

. VARIABLES NOT IN EQUATION

. VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER	LEVEL
. M1	1	.24528	1.00000	.83	1
. B1	2	.42422	1.00000	2.85	1
. S1	3	.84465	1.00000	32.37	1
. M1sqd	5	.06236	1.00000	.05	1
. B1sqd	6	.04707	1.00000	.03	1
. S1sqd	7	-.17424	1.00000	.41	1
. M1B1	8	-.01405	1.00000	.00	1
. M1S1	9	-.06771	1.00000	.06	1
. B1S1	10	-.09125	1.00000	.11	1

STEP NO. 1

VARIABLE ENTERED 3 S1

MULTIPLE R	.8447
MULTIPLE R-SQUARE	.7134
ADJUSTED R-SQUARE	.6914
STD. ERROR OF EST.	632.3545

ANALYSIS OF VARIANCE

	SUM OF SQUARES	DF	MEAN SQUARE	F RATIO
REGRESSION	12941919.	1	.1294192e+08	32.37
RESIDUAL	5198338.5	13	399872.2	

VARIABLES IN EQUATION FOR DE

VARIABLE		COEFFICIENT	STD. ERROR OF COEFF	STD REG COEFF	TOLERANCE	F TO REMOVE	LEVEL
(Y-INTERCEPT		9161.82715)					
S1	3	1115.15405	196.0181	.845	1.00000	32.37	1

. VARIABLES NOT IN EQUATION

. VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER	LEVEL
.					
. M1	1	.45821	1.00000	3.19	1
. B1	2	.79247	1.00000	20.26	1
. M1sqd	5	.11650	1.00000	.17	1
. B1sqd	6	.08792	1.00000	.09	1
. S1sqd	7	-.32548	1.00000	1.42	1
. M1B1	8	-.02625	1.00000	.01	1
. M1S1	9	-.12649	1.00000	.20	1
. B1S1	10	-.17045	1.00000	.36	1

STEP NO. 2

VARIABLE ENTERED 2 B1

MULTIPLE R	.9452
MULTIPLE R-SQUARE	.8934
ADJUSTED R-SQUARE	.8756
STD. ERROR OF EST.	401.4315

ANALYSIS OF VARIANCE

	SUM OF SQUARES	DF	MEAN SQUARE	F RATIO
REGRESSION	16206489.	2	8103245.	50.28
RESIDUAL	1933766.9	12	161147.2	

VARIABLES IN EQUATION FOR DE

VARIABLE		COEFFICIENT	STD. ERROR OF COEFF	STD REG COEFF	TOLERANCE	F TO REMOVE	LEVEL
(Y-INTERCEPT		9161.82715)					
B1	2	560.07813	124.4363	.424	1.00000	20.26	1
S1	3	1115.15405	124.4363	.845	1.00000	80.31	1

. VARIABLES NOT IN EQUATION

. VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER	LEVEL
. M1	1	.75126	1.00000	14.25	1
. M1sqd	5	.19101	1.00000	.42	1
. B1sqd	6	.14416	1.00000	.23	1
. S1sqd	7	-.53365	1.00000	4.38	1
. M1B1	8	-.04303	1.00000	.02	1
. M1S1	9	-.20740	1.00000	.49	1
. B1S1	10	-.27947	1.00000	.93	1

STEP NO. 3

VARIABLE ENTERED 1 M1

MULTIPLE R	.9765
MULTIPLE R-SQUARE	.9536
ADJUSTED R-SQUARE	.9409
STD. ERROR OF EST.	276.7283

ANALYSIS OF VARIANCE

	SUM OF SQUARES	DF	MEAN SQUARE	F RATIO
REGRESSION	17297892.	3	5765964.	75.29
RESIDUAL	842364.06	11	76578.55	

VARIABLES IN EQUATION FOR DE

VARIABLE		COEFFICIENT	STD. ERROR OF COEFF	STD REG COEFF	TOLERANCE	F TO REMOVE	LEVEL
(Y-INTERCEPT		9161.82715)					
M1	1	323.83820	85.7806	.245	1.00000	14.25	1
B1	2	560.07813	85.7806	.424	1.00000	42.63	1
S1	3	1115.15405	85.7806	.845	1.00000	169.00	1

. VARIABLES NOT IN EQUATION

. VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER	LEVEL
.					
. M1sqd	5	.28941	1.00000	.91	1
. B1sqd	6	.21841	1.00000	.50	1
. S1sqd	7	-.00855	1.00000	10.80	1
. M1B1	8	-.06520	1.00000	.04	1
. M1S1	9	-.31423	1.00000	1.10	1
. B1S1	10	-.42344	1.00000	2.18	1

STEP NO. 4

VARIABLE ENTERED 7 S1sqd

MULTIPLE R	.9919
MULTIPLE R-SQUARE	.9839
ADJUSTED R-SQUARE	.9775
STD. ERROR OF EST.	170.7814

ANALYSIS OF VARIANCE

	SUM OF SQUARES	DF	MEAN SQUARE	F RATIO
REGRESSION	17848594.	4	4462149.	152.99
RESIDUAL	291663.00	10	29166.30	

VARIABLES IN EQUATION FOR DE

VARIABLE		COEFFICIENT	STD. ERROR OF COEFF	STD REG COEFF	TOLERANCE	TO REMOVE	LEVEL
(Y-INTERCEPT		9360.52344)					
M1	1	323.83820	52.9391	.245	1.00000	37.42	1
B1	2	560.07813	52.9391	.424	1.00000	111.93	1
S1	3	1115.15405	52.9391	.845	1.00000	443.73	1
S1sqd	7	-286.38565	65.9074	-.174	1.00000	18.88	1

. VARIABLES NOT IN EQUATION

. VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER	LEVEL
. M1sqd	5	-.03776	.85307	.01	1
. B1sqd	6	-.16839	.85307	.26	1
. M1B1	8	-.11081	1.00000	.11	1
. M1S1	9	-.53403	1.00000	3.59	1
. B1S1	10	-.71961	1.00000	9.67	1

STEP NO. 5

VARIABLE ENTERED 10 B1S1

MULTIPLE R	.9961
MULTIPLE R-SQUARE	.9922
ADJUSTED R-SQUARE	.9879
STD. ERROR OF EST.	125.0020

ANALYSIS OF VARIANCE

	SUM OF SQUARES	DF	MEAN SQUARE	F RATIO
REGRESSION	17999626.	5	3599925.	230.39
RESIDUAL	140629.45	9	15625.50	

VARIABLES IN EQUATION FOR DE

VARIABLE		COEFFICIENT	STD. ERROR OF COEFF	STD REG COEFF	TOLERANCE	F TO REMOVE	LEVEL
(Y-INTERCEPT		9360.52344)					
M1	1	323.83820	38.7483	.245	1.00000	69.85	1
B1	2	560.07813	38.7483	.424	1.00000	208.93	1
S1	3	1115.15405	38.7483	.845	1.00000	828.26	1
S1sqd	7	-286.38565	48.2403	-.174	1.00000	35.24	1
B1S1	10	-180.28650	57.9887	-.091	1.00000	9.67	1

. VARIABLES NOT IN EQUATION

. VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER	LEVEL
.					
. M1sqd	5	-.05438	.85307	.02	1
. B1sqd	6	-.24250	.85307	.50	1
. M1B1	8	-.15958	1.00000	.21	1
. M1S1	9	-.76907	1.00000	11.58	1

STEP NO. 6

VARIABLE ENTERED 9 M1S1

MULTIPLE R	.9984
MULTIPLE R-SQUARE	.9968
ADJUSTED R-SQUARE	.9945
STD. ERROR OF EST.	84.7437

ANALYSIS OF VARIANCE

	SUM OF SQUARES	DF	MEAN SQUARE	F RATIO
REGRESSION	18082804.	6	3013801.	419.66
RESIDUAL	57451.914	8	7181.489	

VARIABLES IN EQUATION FOR DE

VARIABLE		COEFFICIENT	STD. ERROR OF COEFF	STD REG COEFF	TOLERANCE	F TO REMOVE	LEVEL
(Y-INTERCEPT		9360.52344)					
M1	1	323.83820	26.2690	.245	1.00000	151.97	1
B1	2	560.07813	26.2690	.424	1.00000	454.58	1
S1	3	1115.15405	26.2690	.845	1.00000	1802.12	1
S1sqd	7	-286.38565	32.7040	-.174	1.00000	76.68	1
M1S1	9	-133.79181	39.3128	-.068	1.00000	11.58	1
B1S1	10	-180.28650	39.3128	-.091	1.00000	21.03	1

. VARIABLES NOT IN EQUATION

. VARIABLE		PARTIAL CORR.	TOLERANCE	F TO ENTER	LEVEL
. M1sqd	5	-.00500	.85307	.05	1
. B1sqd	6	-.37940	.85307	1.18	1
. M1B1	8	-.24967	1.00000	.47	1

.
.
.

***** F LEVELS(4.000, 3.900) OR TOLERANCE INSUFFICIENT FOR FURTHER STEPPING

CORRELATION MATRIX OF REGRESSION COEFFICIENTS

		M1	B1	S1	S1sqd	M1S1	B1S1
		1	2	3	7	9	10
M1	1	1.0000					
B1	2	.0000	1.0000				
S1	3	.0000	.0000	1.0000			
S1sqd	7	.0000	.0000	.0000	1.0000		
M1S1	9	.0000	.0000	.0000	.0000	1.0000	
B1S1	10	.0000	.0000	.0000	.0000	.0000	1.0000

Appendix D: Response Equations and Random Check Points

TABLE D.1

RESPONSE SURFACE EQUATIONS FOR THREE FACTORS

DESIGN	EQUATION
1) BOX - BEHNKEN	Y = 0.4965(M1) + 1.2245(B1) + 1.1802(S1) - .0000375(S1*S1) - .0000665(B1*S1) - 648.3319
2) BOX - DRAPER	Y = 0.4798(M1) + 1.2232(B1) + 1.0847(S1) - .0000318(S1*S1) - .0000667(B1*S1) - 285.6179
3) HYBRID (311A)	Y = 0.5243(M1) + 1.1947(B1) + 0.9613(S1) - .0000251(S1*S1) - .0000602(B1*S1) + 68.4118
4) HYBRID (310+CP)	Y = 0.5002(M1) + 1.327(B1) + 1.163(S1) - .0000352(S1*S1) - .0000774(B1*S1) - 626.1627
5) MIN POINT	Y = 0.4881(M1) + 1.423(B1) + 0.969(S1) - .000025(S1*S1) - .000088(B1*S1) + 210.5637

TABLE D.1 (continued)

RESPONSE SURFACE EQUATIONS FOR THREE FACTORS

DESIGN	EQUATION
6) CENTRAL COMP ROTATABLE	Y = 0.4848(M1) + 1.153(B1) + 1.004(S1) - .0000276(S1*S1) - .0000618(B1*S1) + 124.9285
7) KOSHAL	Y = 0.5453(M1) + 0.6747(B1) + 0.6047(S1) - .0000142(S1*S1) + 2396.592

TABLE D.2

RESPONSE SURFACE EQUATIONS FOR FOUR FACTORS

DESIGN	EQUATION
1) BOX - BEHNKEN	Y = 0.3833(M1) + 0.9461(M2) + 0.9871(B1) + 0.8775(S1) - .0000229(S1*S1) - .0000539(M2*S1) - .0000512(B1*S1) + 1319.9931
2) BOX - DRAPER	Y = 0.3950(M1) + 0.9441(M2) + 0.9879(B1) + 0.8868(S1) - .000024(S1*S1) - .0000527(M2*S1) - .0000503(B1*S1) + 1332.3008
3) HYBRID (416C)	Y = 0.434(M1) + 1.1172(M2) + 1.0793(B1) + 0.7926(S1) - .0000149(S1*S1) - .0000677(M2*S1) - .0000645(B1*S1) + 1129.0738
4) HYBRID (416A+CP)	Y = 0.4394(M1) + 1.0915(M2) + 1.1122(B1) + 0.823(S1) - .0000174(S1*S1) - .0000643(M2*S1) - .0000601(B1*S1) + 1060.9086
5) MIN POINT	Y = 0.4088(M1) + 1.0122(M2) + 1.055(B1) + 0.9707(S1) - .0000265(S1*S1) - .0000582(M2*S1) - .000055(B1*S1) + 810.7629

TABLE D.2 (continued)

RESPONSE SURFACE EQUATIONS FOR FOUR FACTORS

DESIGN	EQUATION
6) CENTRAL COMP ROTATABLE	Y = 0.416(M1) + 0.9609(M2) + 1.0236(B1)
	+ 0.907(S1) − .0000233(S1*S1)
	− .0000536(M2*S1) − .0000512(B1*S1)
	+ 939.3
7) KOSHAL	Y = 0.4087(M1) + 0.4833(M2) + 0.5322(B1)
	+ 0.2406(S1) +5142.2911

TABLE D.3

RESPONSE SURFACE EQUATIONS FOR FIVE FACTORS

DESIGN	EQUATION
1) BOX - BEHNKEN	$Y = 0.6029(M1) + 0.6795(M2)\ 0.7294(B1) + 0.7689(B2) + 1.0363(S1) - .0000295(S1*S1) + 843.9903$
2) BOX - DRAPER (FULL)	$Y = 0.6289(M1) + 0.6776(M2) + 0.7556(B1) + 0.7976(B2) + 1.0942(S1) - .0000329(S1*S1) + 387.9629$
3) BOX - DRAPER (HALF-REPLICATE)	$Y = 0.6037(M1) + 0.6825(M2) + 0.7393(B1) + 0.7846(B2) + 1.0926(S1) - .0000329(S1*S1) + 536.6394$
4) MIN POINT	$Y = 1.2691(M1) + 2.3605(M2) + 0.9176(B1) + 0.938(B2) + 0.6281(S1) - .001015\ (M1*M2) - 640.3131$
5) CENTRAL COMP ROTATABLE	$Y = 0.5011(M1) + 0.6289(M2) + 0.7123(B1) + 0.7545(B2) + 0.9511(S1) - .0000261(S1*S1) + 1615.6741$
6) KOSHAL	$Y = .7010(M1) + 0.7668(M2) + 0.8011(B1) + 0.8332(B2) + 0.9541(S1) - .000024(S1*S1) + 465.7382$

TABLE D.4

RESPONSE SURFACE EQUATIONS FOR SIX FACTORS

DESIGN	EQUATION
1) BOX - BEHNKEN	Y = 0.5358(M1) + 0.6281(M2) + 0.6543(M3) + 0.6921(B1) + 1.3939(B2) + 1.2779(S1) - .0000331(S1*S1) - .0000726(B2*S1) - 642.153
2) BOX - DRAPER (HALF-REPLICATE)	Y = 0.5678(M1) + 0.6446(M2) + 0.6260(M3) + 0.6954(B1) + 1.2237(B2) + 1.2081(S1) - .00003276(S1*S1) - .00005422(B2*S1) - 101.0283
3) HYBRID (628A)	Y = 0.5625(M1) + 0.5981(M2) + 0.6260(M3) + 0.6840(B1) + 1.2294(B2) + 1.0361(S1) - .0000228(S1*S1) - .0000568(B2*S1) + 715.9275
4) HYBRID (628B)	Y = 0.5544(M1) + 0.6061(M2) + 0.6138(M3) + 0.6833(B1) + 1.2290(B2) + 1.0444(S1) - .0000232(S1*S1) - .0000565(B2*S1) + 695.8883
5) MIN POINT	Y = 0.5825(M1) + 0.6558(M2) + 0.5929(M3) + 0.7219(B1) + 1.2001(B2) + 2.3404(S1) - .0000961(S1*S1) - .0000558(B2*S1) - 4816.2332

TABLE D.4 (continued)

RESPONSE SURFACE EQUATIONS FOR SIX FACTORS

DESIGN	EQUATION
6) CENTRAL COMP ROTATABLE	Y = 0.5622(M1) + 0.6062(M2) + 0.6728(M3)
	+ 0.6852(B1) + 1.2372(B2) + 1.0462(S1)
	- .0000235(S1*S1) - .0000569(B2*S1)
	+ 620.122
7) KOSHAL	Y = 0.5837(M1) + 0.6582(M2) + 0.7360(M3)
	+ 0.7033(B1) + 0.9630(B2) + 1.0337(S1)
	- .0000262(S1*S1) - .0000262(B2*S1)
	+ 956.1621

TABLE D.5

RANDOMLY SELECTED CHECK POINTS FOR THREE FACTORS

		M1	B1	S1
	1	1691	1747	6014
	2	1660	1332	10359
	3	2031	2624	9151
	4	2527	1435	7752
	5	2888	2594	8166
	6	1831	1787	6474
	7	1985	2011	7846
	8	2955	1415	7076
	9	2411	1959	6152
	10	1757	2614	8822
	11	2724	2224	9792
	12	2326	2643	11871
	13	2980	1743	11841
	14	1750	2523	9492
	15	1729	2243	9347
	16	2768	2812	8627
	17	2666	1462	8818
	18	2157	2223	8082
	19	2292	2201	9033
	20	1771	2802	7756
***	21	1650	1200	6000
***	22	3000	3000	12000

TABLE D.6

RANDOMLY SELECTED CHECK POINTS FOR FOUR FACTORS

	M1	M2	B1	S1
1	2257	643	1739	6585
2	1665	1443	1400	7407
3	2063	1087	1242	8236
4	2645	1954	2986	9142
5	1881	540	2441	11017
6	2521	1306	1352	8755
7	2496	973	1283	7598
8	2247	1943	1613	7274
9	2779	553	2957	11808
10	2550	1120	1741	11147
11	1739	1056	2371	11175
12	2242	770	1539	11044
13	2608	759	2499	8595
14	2515	628	1278	10883
15	2134	1876	1921	8122
16	2401	1174	1837	11724
17	1800	1819	1373	11811
18	2996	1972	1949	6088
19	1707	1972	1958	8375
20	1712	1870	2027	11200
** 21	1650	500	1200	6000
** 22	3000	2000	3000	12000

TABLE D.7

RANDOMLY SELECTED CHECK POINTS FOR FIVE FACTORS

	M1	M2	B1	B2	S1
1	1691	645	1747	3911	6014
2	1660	535	1332	2462	10359
3	2031	1832	2624	2970	9151
4	2527	562	1435	2822	7752
5	2888	1824	2594	2866	8166
6	1831	1133	1787	2050	6474
7	1985	1670	2011	2832	7846
8	2955	557	1415	2750	7076
9	2411	1656	1959	2652	6152
10	1757	875	2614	2936	8822
11	2295	1252	2235	3613	9208
12	2568	705	1973	2700	6598
13	2073	1977	1368	2588	11543
14	2868	1753	2326	3932	6215
15	1802	1031	1402	2705	6648
16	2108	598	1572	3301	6262
17	1835	1147	1840	2236	8226
18	1873	1281	2344	3994	6801
19	2216	976	2997	2275	8600
20	2138	704	1972	2694	6550
** 21	1650	500	1200	2000	6000
** 22	3000	2000	3000	4000	12000

TABLE D.8

RANDOMLY SELECTED CHECK POINTS FOR SIX FACTORS

	M1	M2	M3	B1	B2	S1
1	2295	1249	561	2371	2244	11147
2	1782	1833	674	2733	2698	11940
3	2316	768	845	2553	2149	11165
4	2944	1912	443	1428	2237	8188
5	1830	1019	912	2320	2333	7243
6	2171	1034	608	1272	2532	8582
7	1691	1720	692	2082	2292	6907
8	1698	953	765	2708	3442	7633
9	1812	1700	514	1234	2135	9257
10	1691	1857	958	2399	2551	11751
11	1660	963	416	2626	3353	6585
12	2031	1323	765	2213	2724	7407
13	2527	517	621	2925	2798	8236
14	2888	1128	704	1359	3910	9142
15	1831	819	548	1415	2253	11017
16	1985	645	554	2201	3079	8755
17	2955	669	943	2907	3265	7598
18	2411	1066	798	2125	2122	7274
19	1757	1944	613	2888	3076	11808
20	2568	1360	863	1539	2291	9235
** 21	1650	500	400	1200	2000	6000
** 22	3000	2000	1000	3000	4000	12000

BIBLIOGRAPHY

1. Box, G.E.P. and D.W. Behnken. "Some New Three Level Designs for the Study of Quantitative Variables." Technometrics, Vol. 2: 455 (1960).

2. Box, G.E.P. and N.R. Draper. "A Basis for the Selection of a Response Surface Design." Journal of American Statistical Association, Vol. 54: 622 (1959).

3. Box, G.E.P. and N.R. Draper. "The Choice of a Second Order Rotatable Design." Biometrika, Vol. 50: 335 (1963).

4. Box, G.E.P. and J.S. Hunter. "Multifactor Experimental Designs for Exploring Response Surfaces." Annals of Mathematical Statistics, Vol. 28: 195 (1957).

5. Box, G.E.P. and K.B. Wilson. "On the Experimental Attainment of Optimum Conditions." Journal of Royal Statistical Society, Series B, Vol. 13: 1 (1951).

6. Box, M.J. and N.R. Draper. "On Minimum Point Second Order Designs." Technometrics, Vol. 16: 613 (1974).

7. Cote, R., A.R. Manson, and R.J. Hader. "Minimum Bias Approximation of a General Regression Model." Journal of American Statistical Association, Vol. 68: 633 (1973).

8. Davies, O.L. Design and Analysis of Industrial Experiments (2nd Edition). New York: Hafner Publishing Co., Inc. (1956).

9. Graney, R. "An Optimization Methodology for Multicriteria Comparison Using Response Surface Methods and Mathematical Programming." Unpublished MS Thesis. AFIT (AU), WPAFB, OH, Mar 1984.

10. Hicks, C.R. Fundamental Concepts in the Design of Experiments (2nd Edition). New York: Holt, Rinehart and Winston (1973).

11. Hill, W.J. and W.G. Hunter. "A Review of Response Surface Methodology: A Literature Survey." Technometrics, Vol. 8: 571 (1966).

12. Kanemasu, H. "A Statistical Approach to Efficient Use and Analysis of Simulation Models," Proceedings of the 42nd Session of the International Statistical Institute, 1979, Manila, Phillipines, 573-601.

13. Karson, M.J. "Design Criterion for Minimum Bias Estimation and Experimental Design for Response Surface." Journal of American Statistical Association, Vol. 65: 1565 (1970).

14. Karson, M.J., A.R. Manson and R.J. Hader. "Minimum Bias Estimation and Experimental Design for Response Surfaces." Technometrics, Vol. 11: 461 (1969).

15. Karson, M.J. and M.L. Spruill. "Design Criteria and Minimum Bias Estimation." Communications in Statistics, Vol. 4: 339 (1975).

16. Lucas J.M. "Which Response Surface Design is Best." Technometrics, Vol. 18: 411 (1976).

17. Manacapilli, T. "A Methodology for Identifying Cost Effective Strategic Force Mixes." Unpublished MS Thesis. AFIT (AU), WPAFB, OH, Dec 1984.

18. Myers, R.H. Response Surface Methodology. Ann Arbor: Edwards Brothers (1971).

19. Neter, J., W. Wasserman, and M.H. Kutner. Applied Linear Statistical Models (Second Edition). Homewood, Illinois: Richard D. Irwin, Inc. (1985).

20. Roquemore, K.G. "Hybrid Designs for Quadratic Response Surfaces." Technometrics, Vol. 18: 419 (1976).

21. Roush, W.B., R.G. Petersen, and G.H. Arscott. "An Application of Response Surface Methodology to Research in Poultry Nutrition." Poultry Science, Vol. 58: 1504 (1979).

22. Smith, P.W. and J.M. Mellichamp. "Multidimensional Parametric Analysis Using Response Surface Methodology and Mathematical Programming as Applied to Military Problems," Proceedings of the Pacific Conference on Operations Research, April 23-28, 1979, Seoul, Korea, Vol. I, 592-615.

23. Steinberg, D.M. and W.G. Hunter. "Experimental Design: Review and Comment." Technometrics, Vol. 26: 71 (1984).

24. Toyomizu, M., Y. Akiba, M. Horiguchi and T. Matsumoto. "Multiple Regression and Response Surface Analysis of the Effects of Dietary Protein, Fat, and Carbohydrate on the Body Protein and Fat in Growing Chicks." Journal of Nutrition, Vol. 112: 886 (1982).

VITA

Captain Bryan K. Ishihara was born on 19 November 1959 in Honolulu, Hawaii. He graduated from Damien Memorial High School in Honolulu, in 1977. He attended the University of Washington, in Seattle, Washington, from 1977 through 1981. He received a Bachelor of Arts in Mathematics and a Bachelor of Arts in Business Administration in June 1981. Upon graduation, he received a commission in the USAF through the ROTC program. His first assignment was as an operations research analyst for the 57th Fighter Weapons Wing, Nellis AFB, Nevada. He worked in the Operations Analysis directorate under the Deputy Commander for Tactics and Test. While at Nellis AFB, he received a Masters Degree in Business Administration from Golden Gate University in August 1983. He served at Nellis AFB until entering the School of Engineering, Air Force Institute of Technology in May 1984.

Permanent Address: 1176 Ala Napunani St
Honolulu, HI 96818

END
DTIC
6-86